

**(Abstract)**

M.Sc Mathematics Programme- Scheme , Syllabus and First Semester Model Question papers(under Choice Based Credit and Semester System (Outcome Based Education system- OBE ) in Affiliated Colleges -Implemented with effect from 2023 admissions - Implemented- Orders issued.

**ACADEMIC C SECTION**

ACAD C/ACAD C4/17258/2023

Dated: 23.08.2023

- Read:-1. U.O No. Acad C2/429/2017 Dated 08.09.2020  
2. U. O No. Acad C1/21246/2019 Dated 07.12.2020  
3. U.O. No. Acad/C1/21246/2019 dated 16.02.2023 ,  
4. U.O. No. Acad/C1/21246/2019 dated 20.04.2023  
5. Minutes of the meeting of the CSMC & Conveners of Ad hoc committee held on 15.06.2023  
6. U.O. No. Acad/C1/21246/2019 dated 09.08.2023  
7. Minutes of the Meeting of the Adhoc committee for M.Sc Mathematics held on 10.08.2023  
8. Syllabus submitted by the Convener, Ad hoc committee for M.Sc Mathematics vide email dated 17.08.2023

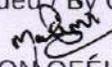
**ORDER**

1. Curriculum Syllabus Monitoring Committee comprising the members of Syndicate was constituted for the Syllabus revision of UG & PG Programmes in Affiliated Colleges, vide paper read (1) above and as per the recommendation of this Committee in its meeting held on 20.11.2020, constitute a sub Committee to prepare the Regulation for PG programmes in Affiliated Colleges vide paper read (2) above.
2. As the reconstitution of Board of Studies of the University is under consideration of the Hon'ble Chancellor, considering the exigency of the matter, Ad hoc Committees were constituted vide paper read (3) above, & it has been modified vide paper read (4) above to revise the Curriculum and Syllabus of PG Programmes in Affiliated Colleges w.e.f 2023-24 academic year.
3. The combined meeting of the Curriculum Syllabus Monitoring Committee & Conveners of Ad hoc committee held on 15.06.2023 at syndicate room discussed in detail the draft Regulation, prepared by the Curriculum Syllabus Monitoring Committee, for the PG programmes under Choice Based Credit and Semester System to be implemented in Affiliated Colleges w.e.f 2023 admission and proposed the different phases of Syllabus revision process such as subject wise workshop , vide paper read (5) above.
4. Revised Regulation for PG programmes under Choice Based Credit and Semester System ( in OBE Outcome Based Education System) was approved by the Vice Chancellor on 05.08.2023 and implemented w.e.f 2023 admission vide paper read (6) above.
5. Subsequently, as per the paper read (7) above, the Ad hoc committee for M.Sc Mathematics programme finalized the Scheme, Syllabus and Model question papers of M.Sc Mathematics programme to be implemented w.e.f 2023 admission.
6. As per the paper read (8) above, the Convener, Ad hoc committee for Mathematics submitted the finalized copy of the Scheme, Syllabus and First semester Model question papers of M.Sc Mathematics programme for implementation w.e.f 2023 admission
7. The Vice Chancellor after considering the matter in detail and in exercise of the powers of the Academic Council conferred under section 11(1) Chapter III of Kannur University Act, 1996 and all other enabling provisions read together with, **accorded sanction to implement the Scheme, Syllabus and First semester Model Question Papers of M.Sc Mathematics programme under Choice Based Credit and Semester System ( in OBE- Outcome Based Education System) in Affiliated Colleges under the University w.e.f 2023 admission , subject to report to the Academic Council**
8. The Scheme, Syllabus and First Semester Model question papers of M.Sc Mathematics programme under Choice Based Credit and Semester System ( in OBE- Outcome Based Education System) in Affiliated Colleges under the University w.e.f 2023 admission is uploaded in the University website.
9. Orders are issued accordingly.

Sd/-  
**Sajesh Kottambrath**  
**Assistant Registrar1**  
For REGISTRAR

To: 1. Principals of Affiliated Colleges offering M.Sc Mathematics Programme  
2. Convener, Curriculum Syllabus Monitoring Committee.  
3. Convener, Adhoc Committee for M.Sc Mathematics Programme

Copy To: 1. The Examination Branch (Through PA to CE)  
2. PS to VC / PA to PVC / PA to R/PA to FO  
3. DR / AR 1 (Acad) /All sections of Academic Branch/Computer Programmer  
4. SF / DF /FC  
5. IT Centre (for uploading on the website)

Forwarded / By Order  
  
SECTION OFFICER





**KANNUR UNIVERSITY**

**കണ്ണൂർ സർവകലാശാല**

Re-accredited by NAAC with 'B++' Grade

**Syllabus for Choice Based Credit and Semester System for  
Post Graduate Programme in Affiliated Colleges - 2023**

# **MSc. MATHEMATICS**

**(2023 Admission onwards)**

**Kannur University**

**Thavakkara, Civil Station P.O. Kannur District.**

**Kerala 670002, India.**

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## **PROGRAMME OUTCOMES**

### **Program Outcomes (POs)**

- PO1.** Advanced Knowledge and Skills: Postgraduate courses aim to provide students with in-depth knowledge and advanced skills related to their chosen field. The best outcome would be to acquire a comprehensive understanding of the subject matter and develop specialized expertise.
- PO2.** Research and Analytical Abilities: Postgraduate programs often emphasize research and analytical thinking. The ability to conduct independent research, analyze complex problems, and propose innovative solutions is highly valued.
- PO3.** Critical Thinking and Problem-Solving Skills: Developing critical thinking skills is crucial for postgraduate students. Being able to evaluate information critically, identify patterns, and solve problems creatively are important outcomes of these programs.
- PO4.** Effective Communication Skills: Strong communication skills, both written and verbal, are essential in various professional settings. Postgraduate programs should focus on enhancing communication abilities to effectively convey ideas, present research findings, and engage in academic discussions.
- PO5.** Ethical and Professional Standards: Graduates should uphold ethical and professional standards relevant to their field. Understanding and adhering to professional ethics and practices are important outcomes of postgraduate education.
- PO6.** Career Readiness: Postgraduate programs should equip students with the necessary skills and knowledge to succeed in their chosen careers. This includes practical skills, industry-specific knowledge, and an understanding of the job market and its requirements.
- PO7.** Networking and Collaboration: Building a professional network and collaborating with peers and experts in the field are valuable outcomes. These connections can lead to opportunities for research collaborations, internships, and employment prospects.
- PO8.** Lifelong Learning: Postgraduate education should instill a passion for lifelong learning. The ability to adapt to new developments in the field, pursue further education, and stay updated with emerging trends is a desirable outcome.

### **Programme Specific Outcomes (PSOs):**

The M.Sc. Mathematics programme's main outcomes are

- PSO1.** Inculcate and develop mathematical aptitude and train students to apply their theoretical knowledge to solve problems
- PSO2.** Develop the knowledge, skills and attitudes necessary to pursue further studies in mathematics
- PSO3.** Develop abstract, logical and critical thinking so that students can reflect critically upon their work and the work of others.
- PSO4.** Appreciate the international dimension of mathematics and its multicultural and historical perspectives.
- PSO5.** Develop in the student the ability to read, follow and appreciate mathematics.
- PSO6.** Train students to communicate mathematical ideas in a lucid and effective manner.
- PSO7.** Have a strong foundation in core areas of Mathematics both pure and applied.
- PSO8.** Communicate mathematical ideas effectively, in writing as well as orally.
- PSO9.** Conduct Professional and Scholarly activities efficiently

## **PROGRAMME STRUCTURE**

**Duration:** The duration of a Post graduate programme shall be four semesters inclusive of days of examinations distributed over a period of two academic years. The odd semesters (1, 3) shall be from June to October and the even semesters (2, 4) shall be from October/November to March. Each semester shall have 90 working days inclusive of days of all examinations. The minimum duration for completion of a two year Post graduate Programme in any subject is four semesters and the maximum period for completion is eight semesters from the date of registration.

**Admission:** Eligibility for admissions and reservation of seats for various First semester (Post Graduate) Programmes shall be according to the rules framed by the University from time to time. There shall be a uniform Academic cum Examinations calendar approved by the University for the registration, conduct and scheduling of examinations, and publication

of results. The Academic cum Examinations Calendar shall be complied with by all colleges and offices, and the Vice Chancellor shall have all powers necessary for this purpose.

**Courses:** The Post graduate programme shall include three types of courses, viz., Core Courses, Elective Courses and Open Elective Courses (including MOOC courses) Parent Department shall offer appropriate elective courses for a specific programme. Open Elective Courses are offered either by the parent department or by any other Department or by via MOOC. Open Elective courses can be opted in third semester preferably having multidisciplinary in nature.

**Project:** There shall be a project work with dissertation to be undertaken by all students. Project and dissertation work is a special course involving application of knowledge in solving/analysing/exploring a real-life situation/problem. The dissertation entails field work, lab work, report, presentation and viva voce. Project with dissertation shall be done under the supervision of a faculty member of the department as per the curriculum design. A candidate may, however, in certain cases be permitted to work on the project in an industrial/ research organisation on the recommendation of the Head of the Department. In such cases, one of the teachers from the department concerned shall be the supervisor/internal guide and an expert from the industry/research organisation concerned shall act as co-supervisor/external guide. Projects shall be submitted in the last week of February in fourth semester. Belated and incomplete projects will not be entertained. Dissertation on project shall be prepared as per the guidelines given as Annexure 1.

**Comprehensive Viva-Voce:** There shall be a comprehensive viva-voce at the end of the programme covering questions from all courses of the programme.

**Credits:** Each course shall have a specified number of credits. The total credits required for successful completion of a four semester programme will be 80. Minimum credits for core course shall be 68. (60 for theory courses and 8 for Dissertation and General Viva-Voce). The number of credits from elective course and Open Elective course is 12 and for Dissertation and General Viva-Voce, the maximum credits shall be 8.

**Attendance:** A student shall be permitted to appear for the semester examination, only if she/he secures not less than 75% attendance in all courses of a semester put together. Relaxation in this attendance shall be given according to the rules and regulations framed by the University from time to time.

**Eligibility to register for examination:** Only those students who are registered for the university examination with eligible attendance (including those under condonable limit) alone are eligible to be promoted to next semester. Students who have attendance in the prescribed limit but could not register for examination are eligible to move to the next semester after availing token registration. The candidates shall apply for token registration within two weeks of the commencement of the next semester. Token registration is allowed only once during the entire programme.

### **COURSE EVALUATION:**

The evaluation scheme for each course shall contain two parts

- a) Continuous Evaluation (CE)
- b) End Semester Evaluation (ESE)

20% weightage shall be given to the Continuous Evaluation (CE) and 80% weightage shall be for the End Semester Evaluation (ESE)

#### **Continuous Evaluation (CE):**

20% of the total marks in each course are for continuous assessment. The continuous evaluation shall be based on a pre-determined transparent system involving written test, assignments, seminar and Viva.

Sl. No.	Components	% of internal marks
1	Two test papers	40
2	Assignments	20
3	Seminar presentation of course study	20
4	Viva voce	20

**Test Paper:** For each course there shall be at least two class tests during a semester. Average mark of best two tests is to be considered for CE. The probable dates of the tests shall be announced at the beginning of each semester. Marks should be displayed on the notice board. Valued answer scripts shall be made available to the students for perusal within 10 working days from the date of the tests.

**Assignment:** Each student shall be required to do two assignments for each course. Assignments after valuation must be returned to the students.

**Seminar:** Each student shall deliver one seminar as an internal component for every course and must be evaluated by the respective teacher in terms of structure.

**Viva voce:** For each course there shall be a viva voce conducted by the faculty.

#### **End Semester Evaluation (ESE)**

End Semester Evaluation carries 80% of total marks. End Semester Evaluation of all semesters will be conducted in centralised valuation camps immediately after the examination.

**Project Evaluation:** Project evaluation shall be conducted at the end of fourth semester as per the following guidelines.

1. Evaluation of the Project Report shall be done under Mark System.
2. The evaluation of the project will be done at two stages:
  - a) Continuous Evaluation (supervising teachers will assess the project and award internal marks)
  - b) End Semester Evaluation (external examiner appointed by the University)
3. Marks secured for the project will be awarded to candidates, combining the Continuous Evaluation and End Semester Evaluation marks.
4. The Continuous Evaluation to End Semester Evaluation components is to be taken in the ratio 1:4. Assessment of different components may be taken as below.

<b>Continuous Evaluation(20% of total)</b>	
<b>Components</b>	<b>Percentage</b>
Punctuality	10
Use of Data	20
Scheme/Organization of Report	40
Presentation and Viva voce	30

<b>End Semester Evaluation (80% of total)</b>	
<b>Components</b>	<b>Percentage</b>
Relevance and structure of the Topic	30
Presentation of facts/ figures/language style/diagrams etc	30
Findings and recommendations	10
Viva-Voce	30

Submission of the Project Report and presence of the student for viva are compulsory for Continuous Evaluation. No marks shall be awarded to a candidate if she/he fails to submit the Project Report for End Semester Evaluation. The student should get a minimum of 40% marks of the aggregate and 40% separately for ESE for pass in the project. There shall be no improvement chance for the Marks obtained in the Project Report. In an instance of inability of obtaining a minimum pass marks as required, the project work shall be re- done and the report may be re-submitted along with subsequent exams through parent department.

Project evaluation and viva-voce shall be conducted by at least two external examiners. 15 minutes time shall be given for the presentation and another 15 minutes for viva voce for each candidates. Maximum number of candidates for evaluation shall not be more than 8 per day.

**Comprehensive Viva-Voce:** There shall be a comprehensive viva voce at the end of the programme covering questions from all courses of the programme. The viva voce shall be conducted by two external examiners. 30 minutes shall be given for each candidates. Maximum number of candidates for this also shall not be more than 8 per day.

**Structure of Question paper for ESE (80 marks)**

PART	No of Questions in the QP	No of Questions to be answered	Marks of each question	Total Marks	%
A	6	5	4	20	25%
B	5	3	7	21	26.25%
C	5	3	13	39	48.75%

Total Number of questions =  $6+5+5 = 16$

Number of questions to be answered =  $5 + 3 + 3=11$

Total Marks in the question paper =  $6 \times 4 + 5 \times 7 + 5 \times 13 = 24 + 35 + 65 = 124$

**Level of questions based on revised Bloom's taxonomy**

PART	Revised Bloom's taxonomy	
	Level	
A	1, 2	Remembering, Understanding
B	6	Creating
C	3, 4, 5	Applying, Analysing, Evaluating

**Time distribution for answering each unit**

PART	No of Questions in the QP	No of Questions to be answered	Approximate time to answer a question (minutes)	Total Time (minutes)
A	6	5	8	40
B	5	3	20	60
C	5	3	25	75

**GRADING:**

Indirect grading system shall be adopted for the assessment of a student's performance in a course (both CE and ESE). Each course is evaluated by assigning marks with a letter grade (A+, A, B, C, D, E and F) to that course by the method of indirect grading. Mark system is followed instead of direct grading for each question. For each course in the semester, letter grade, grade point and percentage of marks are introduced in the indirect grading system with scale as per guidelines given below:

<b>% of Marks (CE+ESE)</b>	<b>Grade</b>	<b>Interpretation</b>
90 and above	A+	Outstanding
80 to below90	A	Excellent
70 to below80	B	Very Good
60 to below 70	C	Good
50 to below 60	D	Satisfactory
40 to below50	E	Pass
Below 40	F	Failure

Evaluation (both CE and ESE) is carried out using Mark system. The grading on the basis of a total CE and ESE marks will be indicated for each course. Each letter grade is assigned a '**Grade point**' (GP) which is a point obtained using the formula:

**Grade Point = (Total marks awarded / Total Maximum marks) x 10.**

'**Credit point**' (CP) of a course is the value obtained by multiplying the grade point (GP) by the credit (C) of the course:

$$\text{CP} = \text{GP} \times \text{C}$$

A minimum of grade point 4 is needed for the successful completion of a course. A candidate securing not less than 40% of aggregate marks of a course with not less than 40% in End Semester Evaluation( ESE )and not less than 10% in Continuous Evaluation (CE ) separately shall be declared to have passed in that course. A minimum of grade point 4 with letter grade E is needed for the successful completion of a course. Appearance for Continuous evaluation (CE) and End Semester Evaluation (ESE) are compulsory and no grade shall be awarded to a candidate if she/he is absent for CE/ESE or both. After the successful completion of a semester, Semester Grade Point Average (SGPA) of a student in that semester is calculated using the formula given below.

**SGPA = Sum of the Credit Points of all courses in a semester / Total Credits in that semester**

**Semester Grade Point Average'** (SGPA) is the value obtained by dividing the sum of credit points obtained by a student in the various courses taken in a semester by the total number of credits in that semester. SGPA determines the overall performance of a student at the end of a semester. For the successful completion of a semester, a student should pass all courses in that semester. However, a student is permitted to move to the next semester irrespective of SGPA obtained. SGPA shall be rounded off to three decimal places.

**The Cumulative Grade Point Average** (CGPA) of the student is calculated at the end of each semester. The CGPA of a student determines the overall academic level of the student in each stage of the programme. CGPA can be calculated by the following formula:

**CGPA = Sum of Credit Points of all completed semesters / Total Credits acquired**

CGPA shall be rounded off to three decimal places.

At the end of the programme, the overall performance of a candidate is indicated by the Overall Grade Point Average. Overall **Grade Point Average (OGPA)** of the student is

calculated at the end of the programme. The OGPA of a student determines the overall academic level the student in a programme and is the criterion for classification and ranking the students. OGPA can be calculated by the following formula

**GPA = Sum of Credit Points obtained in all semesters of the programme / Total Credits (80)**

OGPA shall be rounded off to three decimal places.

An overall letter grade for **OGPA** for the entire programme shall be awarded to a student after completing the entire programme successfully. Overall letter grade based on OGPA and conversion of Grades into classification shall be in the following way.

<b>Grade range OGPA</b>	<b>Overall Letter Grade</b>	<b>Classification</b>
9 - 10	A+	First class with Distinction
8 - 8.999	A	
7 - 7.999	B	First class
6 - 6.999	C	
5 - 5.999	D	Second class
4 - 4.999	E	Pass
Below 4	F	Fail

The Percentage of marks based on OGPA is calculated by multiplying them by 10.

**Percentage in two decimal places = [OGPA in three decimal places] x 10%**

A student who fails to secure a minimum mark for a pass in a course is permitted to write the examination along with the subsequent batch.

**SEMESTER WISE DETAILS:**

<b>SEMESTER -I</b>					
<b>Number of Core Courses: 5</b>					
<b>Sl. No</b>	<b>Course Code</b>	<b>Course Title</b>	<b>Lecture Hours/Week</b>	<b>Marks (Internal+External)</b>	<b>Credits</b>
1	MSMAT01C01	Abstract Algebra	5	100	4
2	MSMAT01C02	Linear Algebra	5	100	4
3	MSMAT01C03	Real Analysis	5	100	4
4	MSMAT01C04	Topology	5	100	4
5	MSMAT01C05	Ordinary Differential Equations	5	100	4
<b>Total credit in core courses</b>			<b>25</b>	<b>500</b>	<b>20</b>
<b>Total credits in Semester - I</b>					<b>20</b>

<b>SEMESTER -II</b>					
<b>Number of Core Courses: 5</b>					
<b>Sl. No</b>	<b>Course Code</b>	<b>Course Title</b>	<b>Lecture Hours/Week</b>	<b>Marks (Internal+External)</b>	<b>Credits</b>
1	MSMAT02C06	Advanced Abstract Algebra	5	100	4
2	MSMAT02C07	Measure Theory	5	100	4
3	MSMAT02C08	Advanced Real Analysis	5	100	4
4	MSMAT02C09	Advanced Topology	5	100	4
5	MSMAT02C10	PDE and Integral Equations	5	100	4
<b>Total credit in core courses</b>			<b>25</b>	<b>500</b>	<b>20</b>
<b>Total credits in Semester - II</b>					<b>20</b>

<b>SEMESTER –III</b>					
<b>Number of Core Courses: 3</b>					
<b>Sl. No</b>	<b>Course Code</b>	<b>Course Title</b>	<b>Lecture Hours/Week</b>	<b>Marks (Internal+External)</b>	<b>Credits</b>
1	MSMAT03C11	Functional Analysis	5	100	4
2	MSMAT03C12	Complex Analysis	5	100	4
3	MSMAT03C13	Differential Geometry	5	100	4
<b>Total credit in core courses</b>			<b>15</b>	<b>300</b>	<b>12</b>
<b>Number of Core Elective Courses: 1</b>					
4	MSMAT03E01/ 02/03/04	Elective Course 1	5	100	4
<b>Total credit in core elective courses</b>			<b>5</b>	<b>100</b>	<b>4</b>
<b>Number of Open Elective Courses: 1</b>					
5	MSMAT03O 01/ 02/03/04/05/06	Open Elective Course 1	5	100	4
<b>Total credit in open elective courses</b>			<b>5</b>	<b>100</b>	<b>4</b>
<b>Total credits in Semester – III</b>					<b>20</b>

<b>SEMESTER –IV</b>					
<b>Number of Core Courses: 4</b>					
<b>Sl. No</b>	<b>Course Code</b>	<b>Course Title</b>	<b>Lecture Hours/Week</b>	<b>Marks (Internal+External)</b>	<b>Credits</b>
1	MSMAT04C14	Operator Theory	5	100	4
2	MSMAT04C15	Complex Function Theory	5	100	4
3	MSMAT04C16	Project/ Dissertation	10	100	4
4	MSMAT04C17	Comprehensive Viva - Voce	--	100	4
<b>Total credit in core courses</b>			<b>20</b>	<b>400</b>	<b>16</b>
<b>Number of Core Elective Courses: 1</b>					

3	MSMAT04E05/ 06/07/08	Elective Course 1	5	100	4
	<b>Total credit in core elective courses</b>		<b>5</b>	<b>100</b>	<b>4</b>
	<b>Total credits in Semester - IV</b>				<b>20</b>

**ELECTIVE COURSES:**

<b>Core Elective Courses : III Semester</b>	
<b>COURSE CODE</b>	<b>COURSE TITLE</b>
MSMAT03E01	Number Theory
MSMAT03E02	Calculus of Variations
MSMAT03E03	Algebraic Topology
MSMAT03E04	Numerical Analysis and computing

<b>Open Elective Courses: III Semester</b>	
<b>COURSE CODE</b>	<b>COURSE TITLE</b>
MSMAT03O01	Graph Theory
MSMAT03O02	Discrete Mathematics
MSMAT03O03	Operations Research
MSMAT03O04	Fuzzy Mathematics
MSMAT03O05	Coding Theory
MSMAT03O06	Automata and Formal Languages

<b>Core Elective Courses : IV Semester</b>	
<b>COURSE CODE</b>	<b>COURSE TITLE</b>
MSMAT04E05	Commutative Algebra
MSMAT04E06	Fourier and Wavelet Analysis
MSMAT04E07	Probability Theory
MSMAT04E08	Advanced Differential Geometry

## **SEMESTER-I**

### **MSMAT01C01: Abstract Algebra**

**Course Objective:** To provide a first approach to the subject of algebra, which is one of the basic pillars of modern mathematics and to gain knowledge in basic group theory and ring theory which are essential for further study.

**Course Learning outcome:** After successful completion of the course, student will be able to understand the basic algebraic structures such as group theory and ring theory.

#### **Unit I**

Direct Products and finitely generated Abelian Groups, Group Action on a Set  
(Chapter 2: Section 9; Chapter 3: Section 14)

#### **Unit II**

Sylow Theorems, Applications of Sylow Theorems, Field of Quotients of the Integral Domain.  
(Chapter 4: Section 17; Chapter 6: Section 26)

#### **Unit III**

Isomorphism Theorems, Series of Groups, Free Abelian Groups  
(Chapter 4: Section 16, 18, 19)

#### **Unit IV**

Ring of Polynomials, Factorization of Polynomials over a Field, Homomorphisms and Factor Rings, Prime and Maximal Ideals  
(Chapter 6: Section 27, 28, 30, 31)

**Text Book:** John. B. Fraleigh, Neal E. Brand, *A First Course in Abstract Algebra* (Eighth Edition), Pearson, 2021

#### **References:**

1. Joseph A. Gallian, *Contemporary Abstract Algebra*, Narosa, 1999
2. I. N. Herstein, *Topics in Algebra*, Wiley India Pvt. Ltd, 2006
3. M. Artin, *Algebra* (Second Edition) Addison Wesley, 2010
4. David S. Dummit, *Abstract Algebra* (Third Edition), Wiley India, 2011
5. D. S. Malik, John. N. Merdson, M. K. Sen, *Fundamentals of Abstract Algebra*, McGraw-hill Publishing Co., 1996
6. Allan Clark, *Elements of Abstract Algebra*, Dover Publications, 1984
7. David M. Burton, *A First course in Rings and Ideals*, Addison-Wesley Educational Publishers, 1970

## MSMAT01C02: Linear Algebra

**Course Objective:** To understand the basic ideas of Vector spaces, Linear transformations, Decomposition of Linear operators and Inner product spaces which are essential to learn advanced level mathematics.

**Course Learning outcome:** After successful completion of the course, student will be able to understand the basic linear algebra- vector space, linear transformations and inner product spaces.

### Unit I

**Linear Transformations:** Linear Transformations, The algebra of Linear Transformations, Isomorphism, Representation of Transformation by Matrices  
(Chapter 3: Sections 3.1 to 3.4,)

### Unit II

Linear Functionals, The Double Dual, The Transpose of a Linear Transformation.

**Elementary Canonical Forms:** Introductions, Characteristic values  
(Chapter 3: Sections 3.5 to 3.7 Chapter 6: Section 6.1 and 6.2)

### Unit III

**Elementary Canonical Forms:** Annihilating Polynomials, Invariant Subspace, Simultaneous Triangulations & Simultaneous Diagonalisation, Direct Sum Decompositions, Invariant Direct Sums

(Chapter 6: Sections 6.3 to 6.7)

### Unit IV

**Elementary Canonical Forms:** The Primary Decomposition Theorem.

Cyclic Subspaces and Annihilators, Cyclic Decomposition and Generalized Cayley-Hamilton Theorem (Proof of Cyclic Decomposition Theorem is excluded)

**Inner Product Spaces:** Inner Products, Inner Product Spaces,

(Chapter 6: Section 6.8; Chapter 7: Sections: 7.1, 7.2 (up to Theorem 4), Chapter 8: Sections 8.1, 8.2)

**Text Book:** Kenneth Hoffman & Ray Kunze: *Linear Algebra* (Second Edition), Prentice- Hall of India Pvt. Ltd, 2015.

### References:

1. Stephen H Friedberg, Arnold J Insel and Lawrence E Spence, *Linear Algebra* (Fourth Edition), Prentice Hall, 2015.
2. Sheldon Axler, *Linear Algebra Done Right* (Third Edition), Springer, 2015
3. Martin Anthony and Michele Harvey, *Linear Algebra: Concepts and Methods*, Cambridge University Press, 2012
4. S. Kumaresan, *Linear Algebra: A Geometric Approach*, PHI Learning Pvt. Ltd., 2000

5. Serge A Lang, *Linear Algebra* (Third Edition), Springer, 2004.
6. Paul R Halmos, *Finite-Dimensional Vector Spaces*, Springer 1974.
7. Michael Artin, *Algebra* (Second Edition) Addison Wesley, 2010

### **MSMAT01C03: Real Analysis**

**Course objective:** To develop basic concepts like limit, convergence, differentiation and Riemann integral. Convergence of functions.

**Course Learning outcome:** After successful completion of the course, student will be able to understand the basic real analysis- convergence, differentiation and integration.

#### **Unit I**

Basic Topology-Finite, Countable and uncountable Sets, Metric spaces, Compact Sets, Perfect Sets, Connected Sets. (Chapter 2)

#### **Unit II**

Continuity-Limits of function, Continuous functions, Continuity and compactness, continuity and connectedness, Discontinuities, Monotonic functions, Infinite limits and Limits at infinity. (Chapter 4)

#### **Unit III**

Differentiation, Derivative of a real function. Mean value theorems, Continuity of derivatives. L Hospital's rule. Derivatives of higher order. Taylor's theorem. Differentiation of vector valued functions. (Chapter 5)

#### **Unit IV**

Reimann – Stieltjes integral. Definition and existence of the integral. Integration and differentiation. Integration of vector – valued functions. Rectifiable curves. (Chapter 6)

**Text Book:** Walter Rudin, *Principles of Mathematical Analysis* (Third Edition), McGraw Hill, 1976.

#### **References:**

1. T. M. Apostol, *Mathematical Analysis* (Second Edition), Narosa, 2002
2. R. G. Bartle, *The Elements of Real Analysis*(Second Edition), Wiley International, 1975
3. G. F. Simmons, *Introduction to Topology and Modern Analysis*, McGraw- Hill, 2017
4. Charles Chapman Pugh, *Real Mathematical Analysis*, Springer, 2010
5. Sudhir R. Ghorpade and Balmohan V. Limaye, *A Course in Calculus and Real Analysis*, Springer, 2006
6. R. G. Bartle and D. R Sherbert, *Introduction to Real Analysis*, John Wiley Bros., 1982
7. L. M Graves, *The Theory of Functions of a Real Variable*, Tata McGraw- Hill, 1978
8. M. H Protter and C.B Moray, *A First course in Real Analysis*, Springer, 1977

## **MSMAT01C04: Topology**

**Course Objective:** To present an introduction to the theory of topology, a powerful tool for understanding other branches of mathematics.

**Course Learning outcome:** After successful completion of the course, student will be able to understand the topological spaces, continuous functions and connected spaces.

### **Unit I**

Topological spaces, Basis for a topology, The order topology, The product topology (finite).  
(Chapter 2: Sections 12 to 15)

### **Unit II**

The subspace topology, Closed sets and limit points, Continuous functions.  
(Chapter 2: Sections 16 to 18)

### **Unit III**

The product topology, The metric topology (Theorem 20.4 & Theorem 20.5 statement only),  
The metric topology (continued), The Quotient Topology.  
(Chapter 2: Sections 19 to 22)

### **Unit IV**

Connected spaces, Connected subspace of the real line, Components and local connectedness.  
(Chapter 3: Sections 23 to 25)

**Text Book:** James. R. Munkres, *Topology- A First Course*, Pearson India (Second Edition), 2014.

### **References:**

1. C. Wayne Patty, *Foundations of Topology*, Joes and Bartlett, 2010.
2. K. Parthasarathy, *Topology - An invitation*, Springer 2022
3. G.F. Simmons, *Introduction to Topology and Modern Analysis*, McGraw Hill, 2017.
4. K. D. Joshi, *Introduction to General Topology*, New age International, 1983
5. Singer and J. A. Thorpe, *Lecture Notes on Elementary Topology and Geometry*, Springer, 1967.
6. J. L. Kelley, *General Topology*, Springer, 1975.
7. Stephen Willard, *General Topology*, Dover Books, 1970.

## MSMAT01C05: Ordinary Differential Equations

**Course Objective:** To gain knowledge on the basic differential equations at the heart of analysis which is a dominant branch of mathematics for 300 years. This subject is the natural purpose of the primary calculus and the most important part of mathematics for understanding physics.

**Course Learning outcome:** After successful completion of the course, student will be able to understand the basics of differential equations and the method of solving them.

### Unit I

Introduction: A Review of Power Series, Series Solutions of First Order Equations, Second Order Linear Equations. Ordinary Points, Regular Singular Points, Regular Singular Points (Continued), Gauss's Hyper Geometric Equation, The Point at Infinity. (Chapter 5: Sections 26 to 32)

### Unit II

Legendre Polynomials, Properties of Legendre Polynomials, Bessel Functions, The Gamma Function, Properties of Bessel functions (Chapter 8: Sections 44 to 47)

### Unit III

Oscillations and the Sturm Separation Theorem, The Sturm Comparison Theorem, General Remarks on Systems, Linear Systems, Homogeneous Linear Systems with Constant Coefficients. (Chapter 4: Sections 24 and 25; Chapter 10: Sections 54 to 56)

### Unit IV

The Method of Successive Approximations, Picard's Theorem, Systems, The Second Order Linear Equation (Chapter 13: Sections 69 to 71)

**Text Book:** G.F Simmons, *Differential Equations with Historical Notes* (Third Edition), CRC Press-Taylor and Francis Group, 2017.

### References:

1. G. Birkoff and G. C Rota, *Ordinary Differential Equations* (Fourth Edition), Wiley and Sons, 1978.
2. E. A Coddington, *An Introduction to Ordinary Differential Equations*, Prentice Hall of India, 1974.
3. P. Hartmon, *Ordinary Differential Equations*, Society for Industrial and applied., 1987
4. Chakraborti, *Elements of Ordinary Differential Equations and Special Functions*, Wiley Eastern, 1990
5. L.S Poutrigardian, *A Course in Ordinary Differential Equations*, Hindustan Publishing Corp., 1967
6. S.G Deo and V.Raghavendra, *Ordinary Differential Equations and Stability Theory*, Tata McGraw Hill, 1967
8. V. I. Arnold, *Ordinary Differential Equations*, MIT Press Cambridge, 1981
9. A. K. Nandakumaran, P. S. Dutty and Raju K George, *Ordinary Differential Equations: Principles and Applications*, Cambridge University Press, 2017

## SEMESTER-II

### **MSMAT02C06: Advanced Abstract Algebra**

**Course Objective:** The aim of this course is to learn advances in algebra and Galois Theory.

**Course Learning outcome:** After successful completion of the course, student will be able to understand some topics in algebra, including Galois theory.

#### **Unit I**

Unique Factorization Domains, Euclidean Domains, Gaussian Integers and Multiplicative Norms (Chapter 7: Sections 34 to 36)

#### **Unit II**

Introduction to Extension Fields, Algebraic Extensions, Geometric Constructions, Finite Fields (Chapter 8: Sections 39 to 42)

#### **Unit III**

Introduction to Galois Theory , Splitting Fields , The Isomorphism Extension Theorem (Chapter 9: Sections 43 and 44 ).

#### **Unit IV**

Separable Extensions. Galois Theory (Chapter 9: Sections 45 and 46).

**Text Book:** John. B. Fraleigh, Neal E. Brand, *A First Course in Abstract Algebra* (Eighth Edition), Pearson, 2021

#### **References:**

1. Joseph A. Gallian, *Contemporary Abstract Algebra*, Narosa, 1999
2. I. N. Herstein, *Topics in Algebra*, Wiley India Pvt. Ltd, 2006
3. M. Artin, *Algebra* (Second Edition) Addison Wesley, 2010
4. David S. Dummit, *Abstract Algebra* (Third Edition), Wiley India, 2011
5. D. S. Malik, John. N. Merdson, M. K. Sen, *Fundamentals of Abstract Algebra*, McGraw-hill Publishing Co., 1996
6. Allan Clark, *Elements of Abstract Algebra*, Dover Publications, 1984
7. David M. Burton, *A First course in Rings and Ideals*, Addison-Wesley Educational Publishers, 1970

## MSMAT02C07: Measure Theory

**Course Objective:** The aim of this course is to learn the Measure Theory, Lebesgue Integration and related Concepts

**Course Learning outcome:** After successful completion of the course, student will be able to understand some topics in measure theory, Lebesgue integration.

### Unit I

**Measure on the real line:** Lebesgue Outer measure, Measurable sets, Regularity, Measurable Functions, Borel and Lebesgue Measurability (Including Theorem 17), (Chapter 2: Sections 2.1 to 2.5).

### Unit II

**Integration of functions of a Real Variable:** Integration of Non-negative Functions, Integration of functions of a Real Variable, The general Integral, Riemann and Lebesgue Integrals.

(Chapter 3: Sections 3.1, 3.2 and 3.4)

### Unit III

**Abstract Measure Space:** Measures and Outer measures, extension of measure, Uniqueness of the extension, Measure Spaces

(Chapter 5: Sections 5.1 to 5.3, 5.5)

### Unit IV

**Abstract Measure Space:** Integration with respect to a Measure, **Inequalities and the  $L^p$  Spaces:** The  $L^p$  Spaces, The inequalities of Holder and Minkowski, Completeness of  $L^p(\mu)$

(Chapter 5; Section 5.6; Chapter 6: Sections 6.1, 6.4 and 6.5)

**Text Book:** G De Barra, *Measure Theory and Integration*. (Second Edition), New Age International Pvt. Ltd., 2003.

### Reference:

1. Walter Rudin, *Real and Complex Analysis* (Third Edition), Tata McGraw Hill, 2017
2. H. L Royden, P M Fitzpatrick, *Real Analysis*, Pearson, Fourth Edition, Pearson, 2015
3. R. G. Bartle, *The Elements of Integration and Measure Theory*, John Wiley and Sons, 1995
4. P.R Halmos, *Measure Theory*, Springer, 1976
5. A. E Taylor, *General Theory of Functions and Integrations*, Dover Publications, 2010
6. Inder K. Rana, *An Introduction to Measure and Integration*, Narosa Publishing House, 1997
7. M.Thamban Nair, *Measure and Integration: A First Course*, CRC Press, 2019

## **MSMAT02C08: Advanced Real Analysis**

**Course Objective:** The aim of this course is to learn the real analysis in advanced level. The course provides the basis for further studies within functional analysis, topology and function theory.

**Course Learning outcome:** After successful completion of the course, student will be able to understand uniform convergence and functions of several variables.

### **Unit I**

Sequence and series of Functions: Discussion of Main Problem, Uniform Convergence, Uniform Convergence and Continuity. (Chapter 7: Sections 7.1 to 7.15)

### **Unit II**

Uniform Convergence and Integration, Uniform Convergence and Differentiation, Equicontinuous Families of Functions, The Stone-Weierstrass Theorem (Chapter 7: Sections 7.16 to 7.33)

### **Unit III**

Some Special Functions: Power Series, The Exponential and Logarithmic Functions, The Trigonometric Functions, The Algebraic Completeness of the Complex Field, Fourier Series, The Gamma Function (Chapter- 8)

### **Unit IV**

Functions of Several Variables: Norm and invertible linear Operators, Differentiation, The Contraction Principle, The Inverse Function Theorem, The Implicit Function Theorem (Chapter 9: Sections 9.6 to 9.29)

**Text Book:** Walter Rudin, *Principles of Mathematical Analysis* (Third Edition) McGraw Hill, 1986.

### **Reference:**

1. T.M. Apostol, *Mathematical Analysis* (Second Edition), Narosa, 2002
2. R. G. Bartle, *The Elements of Real Analysis*(Second Edition), Wiley International, 1975
3. G.F. Simmons, *Introduction to Topology and Modern Analysis*, McGraw Hill, 2017
4. Charles Chapman Pugh, *Real Mathematical Analysis*, Springer, 2010
5. S. R. Ghorpade and B. V. Limaye, *A Course in Calculus and Real Analysis*, Springer, 2006
6. R. G Bartle and D. R Sherbert, *Introduction to Real Analysis*, John Wiley Bros., 1982
7. L. M Graves, *The Theory of Functions of a Real Variable*, Tata McGraw Hill, 1978
8. M. H Protter and C.B Moray, *A First course in Real Analysis*, Springer, 1977

## MSMAT02C09: Advanced Topology

**Course Objective:** To learn advanced level theory of topology, a powerful tool for understanding other branches of mathematics.

**Course Learning outcome:** After successful completion of the course, student will be able to understand Compactness, Separation Axioms and classical Theorems in topology such as Urysohn Lemma, Urysohn Metrization theorem, Tietze Extension, Tychonoff Theorem and Stone – Cech Compactification

### Unit I

Well ordered sets, Compact spaces, Compact subset of the real line, Limit point compactness, Local compactness (Chapter 1: Section 10, Chapter 3: Sections 26, 27, 28 and 29)

### Unit II

The Countability Axioms, The separation Axioms, Normal Spaces. (Chapter 4: Section 30, 31, 32)

### Unit III

The Urysohn Lemma, The Urysohn Metrization theorem, The Tietze Extension Theorem (Chapter 4: Section 33, 34, 35)

### Unit IV

The Tychonoff Theorem, The Stone – Cech Compactification (Chapter 5: Section 37, 38)

**Text Book:** James. R. Munkres, *Topology- A First Course*, Pearson India (Second Edition), 2015.

### References:

1. C. Wayne Patty, *Foundations of Topology*, Joes and Bartlett, 2010.
2. K. D. Joshi, *Introduction to General Topology*, New age International, 1983
3. G.F. Simmons, *Introduction to Topology and Modern Analysis*, McGraw Hill, 2017.
4. Singer and J. A. Thorpe, *Lecture Notes on Elementary Topology and Geometry*, Springer, 1967.
5. J. L. Kelley, *General Topology*, Springer, 1975.
6. Stephen Willard, *General Topology*, Dover Books, 1970.

## **MSMAT02C10: PDE and Integral Equations**

**Course Objective:** To learn about partial differential equations, integral equations and finding solutions using various techniques

**Course Outcome:** Upon the successful completion of the course students will learn techniques to solve first order PDE and analyse the solution to get information about the parameters involved in the model and get an idea about Integral equations

### **Unit I**

**First-order equations:** Introduction, Quasilinear equations, The method of characteristics, Examples of the characteristics method, The existence and uniqueness theorem, The Lagrange method, General nonlinear equations, Exercises.

(Chapter 2: Sections 2.1 to 2.6, 2.9, 2.10 from Text 1)

### **Unit II**

**Second-order linear equations in two independent variables:** Classification, Canonical form of hyperbolic equations, Canonical form of parabolic equations, Canonical form of elliptic equations

**The one-dimensional wave equation:** Introduction, Canonical form and general solution, The Cauchy problem and d'Alembert's formula, Domain of dependence and region of influence, The Cauchy problem for the nonhomogeneous wave equation

(Chapters 3 and 4 from Text 1)

### **Unit III**

**The method of separation of variables:** Introduction, Heat equation: homogeneous boundary condition, Separation of variables for the wave equation, Separation of variables for nonhomogeneous equations, The energy method and uniqueness, Further applications of the heat equation.

**Elliptic equations:** Introduction, Basic properties of elliptic problems, The maximum principle, Applications of the maximum principle, Green's identities, The maximum principle for the heat equation, Separation of variables for elliptic problems, Poisson's formula.

(Chapters 5 and 7 from Text 1)

### **Unit IV**

**Integral Equations:** Introduction, Relations between differential and integral equations, The Green's functions, Fredholm equations with separable kernels, Illustrative examples, Hilbert-Schmidt Theory, Iterative methods for solving Equations of the second kind. The Neumann Series, Fredholm Theory [Chapter 3: Sections 3.1 to 3.3, 3.6 to 3.11 from the Text 2]

### **Text Books:**

1. YehudhaPinchover and Jacob Rubienstein, *An Introduction to Partial Differential Equations*, Cambridge University Press, 2005
2. F. BHildebrand, *Methods of Applied Mathematics*(Second Edition), Prentice Hall, 1972.

**References:**

1. Amaranath T, *Partial Differential Equations*, Narosa, 1997.
2. A. Chakrabarti, *Elements of ordinary Differential Equations and special functions*, Wiley Eastern Ltd, 1990
3. E.A. Coddington, *An Introduction to Ordinary Differential Equations*, Printice Hall of India, 1974
4. R. Courant and D. Hilbert, *Methods of Mathematical Physics-Vol I*, Wiley Eastern Reprint, 1975

## SEMESTER-III

### MSMAT03C11: Functional Analysis

**Course Objective:** To enable the students to connect topology, linear algebra and analysis together to get into the theory of functional analysis.

**Course Learning outcome:** After successful completion of the course, student will be able to bring together the theories of linear algebra, topology and analysis and get acquainted with the basic theories of functional analysis.

#### Unit I

**Complete Metric spaces:** Definition of Metric space, Sequence space  $l^\infty$ , Function space  $C[a,b]$ . Examples of Metric Spaces: Sequence space  $s$ . Space  $B(A)$  of bounded functions. Space  $l^p$ , Hilbert sequence space  $l^2$ , Hölder and Minkowski inequalities for sums. Definition of Dense set and separable space. Non separability of space  $l^\infty$  and separability of Space  $l^p$ ,  $1 \leq p < +\infty$ . Definition of Cauchy sequence and completeness. Completeness of  $l^\infty$ . Completeness of  $c$ , Completeness of  $l^p$ ,  $1 \leq p < +\infty$ . Completeness of space  $C[a,b]$ . (All Proofs of Statements, Results and Examples of these topics from Chapter 1 is excluded)

**Normed space and Banach space:** Normed space and Banach space, Further Properties of Normed spaces, Finite dimensional normed spaces and subspaces, compactness and finite dimension.

(Chapter 1, Section 1.1-1, 1.1-6, 1.1-7, 1.2-1, 1.2-2, 1.2-3, 1.3-5, 1.3-9, 1.3-10, 1.4-3, 1.5-2, 1.5-3, 1.5-4, 1.5-5 Statements only, Proof excluded).

(Chapter 2: Sections 2.2 to 2.5)

#### Unit II

**Linear operators and linear functionals :** Linear operators, Bounded and continuous linear operators, linear functionals. Linear operators and functionals on finite dimensional spaces. Normed Spaces of Operators, Dual Space.

(Chapter 2: Section 2.6 to 2.10)

#### Unit III

**Inner Product spaces and Hilbert spaces:** Inner Product space. Hilbert space. Properties of Inner Product Spaces, Orthogonal Complements and Direct Sums, Orthonormal Sets and Sequences, Series Related to Orthonormal Sequences and Sets. Total Orthonormal Sets and Sequences.

(Chapter 3: Section 3.1 to 3.6).

#### Unit IV

**Functionals and Operators on Hilbert spaces:** Representation of Functionals on Hilbert spaces, Hilbert-Adjoint operator, Self adjoint, unitary and normal operators.

**Fundamental Theorems for Normed and Banach Spaces:** Zorn's lemma, Hahn-Banach theorem, Hahn-Banach theorem for complex vector spaces and normed spaces.

(Chapter 3: Section 3.8 to 3.10; Chapter 4: Section 4.1 to 4.3).

**Text Book:** Erwin Kreyszig, *Introductory Functional Analysis with Applications*, John Wiley, 1991

#### References:

1. Balmohan V Limaye, *Functional Analysis* (Second Edition), New Age International Publishers, 1996
2. M.T Nair, *Functional Analysis: A First Course*; Prentice Hall of India, 2021
3. Walter Rudin, *Functional Analysis*, McGraw-Hill Inc., US, 1978

4. B.Choudhary, Sudarsan Nanda, *Functional Analysis with Applications* (Second edition), New Age International (P) Ltd., 2003
5. Walter Rudin, *Real and Complex Analysis*, McGraw Hill Education, 2017
6. J.B Conway, *A Course in Functional Analysis*, Springer, 2010

### **MSMAT03C12: Complex Analysis**

**Course Objective:** To enable the students to develop basic complex analysis techniques for advanced level complex functiona theory

**Course Learning outcome:** After successful completion of the course, student will study Cauchys theorems, residue integration and space of analytic and meromorphic functions.

#### **Unit I**

Power Series representation of analytic functions, Zeros of an analytic function, The index of a closed curve, Cauchy's theorem and Integral formula, The homotopic version of Cauchy's Theorem and simple connectivity  
(Chapter 4: Sections 2 to 6)

#### **Unit II**

Counting zeros, the Open Mapping Theorem, Goursat's Theorem. Classification of Singularities, Residues  
(Chapter 4: Sections 7 and 8; Chapter 5: Sections 1 and 2)

#### **Unit III**

The Argument Principle, The Maximum Principle, Schwarz's Lemma  
(Chapter 5: Section 3; Chapter 6: Sections 1 and 2)

#### **Unit IV**

The space of continuous functions  $C(G, \Omega)$ (Omit proof of Arzela-Ascoli Theorem), Spaces of analytic functions, The Reimann Mapping Theorem  
(Chapter 7: Sections 1, 2 and 4 (Omit Theorem 1.23))

**Text Book:** John B Conway, *Functions of One Complex Variable* (Second Edition), Springer, 1995

#### **Reference:**

1. Louis Pennise, *Elements of Complex Variable*, Half, Richart& Winston, 1976
2. H. Silverman, *Complex Variable*, Haughton Miffin Complex, Boston, 1975.
3. Walter Rudin, *Real and Complex Analysis* (Third Edition) McGraw Hill International Edition, 1967
4. E. T Copson, *An Introduction to the Theory of a Complex Variables*, Oxford University Press, 1974.
5. Lars V. Ahlfors, *Complex Analysis* (Third Edition), McGraw Hill Education, 2017
6. Theodore W. Gamelin, *Complex Analysis*, Springer, 2001

## **MSMAT03C13: Differential Geometry**

**Course Objective:** The course gives an introduction to the elementary concepts of differential geometry using the calculus of vector fields so that the students also attain a deep understanding of several variables calculus.

**Course Learning outcome:** After successful completion of the course, student will be able to understand the basics of differential geometry and several variable calculus.

### **Unit I**

Graphs and Level Sets, Vector Fields, The Tangent Space, Surfaces, Vector Fields on Surfaces; Orientation (Chapters 1 to 5)

### **Unit II**

The Gauss Map, Geodesics, Parallel Transport (Chapters 6 to 8)

### **Unit III**

Weingarten Map, Curvature of Plane Curves Arc Length and Line Integrals, (Chapters 9 to 11)

### **Unit IV**

Curvature of Surfaces, Parametrized Surfaces. (Chapters 12 and 14)

**Text Book:** J. A. Thorpe, *Elementary Topics in Differential Geometry*, Springer- Verlag, 1979.

### **References:**

1. Andrew Priestley, *Elementary Differential Geometry*, Springer, 2001
2. Sean Dineen, *Multivariate Calculus and Geometry*, Springer, 1998
3. D. J. Struik, *Classical Differential Geometry* (Second Edition), Dover, 1988
4. E. Kreyszig, *Introduction to Differential Geometry and Riemannian Geometry*, Univ. of Toronto Press, 1969
5. Manfredo P. Do Carmo, *Differential Geometry of Curves and Surfaces* (Second Edition), Dover Books, 2016
6. Barrot O Neill, *Elementary Differential Geometry* (Revised Second Edition), Academic Press, 1997

## Core Elective Papers (Third Semester)

### MSMAT03E01: Number Theory

**Course Objective:** To enable the students to develop basic concepts in Number Theory.

**Course Learning outcome:** After successful completion of the course, student will study the basics of both Analytic and Algebraic Number Theory.

#### Unit I

**Arithmetical Functions and Dirichlet multiplication:** Introduction- The Mobius function  $\mu(n)$ -The Euler totient function  $\phi(n)$  -The relation connecting  $\mu$  and  $\phi$  -the product formula for  $\phi(n)$  -The Dirichlet product of arithmetical functions- Dirichlet inverses and Mobius inversion formula- The Mangolt function  $\Lambda(n)$  -Multiplicative functions- Multiplicative functions and Dirichlet multiplication- The inverse of a completely multiplicative function- Liouville's function  $\lambda(n)$ - The divisor function  $\sigma_\alpha(n)$  .

(Text 1: Section 2.1 to 2.13)

**Congruences:** Residue classes and complete residue system- Linear Congruences-Reduced residue system and the Euler- Fermat theorem- Polynomial congruences modulo  $p$  and Langrange's theorem- Applications of Langrange's theorem- Simultaneous linear congruences and Chinese Remainder theorem- Applications of Chinese remainder theorem- Polynomial congruences with prime power moduli.

(Text 1: Section 5.2 to 5.9)

#### Unit II

**Quadratic Residues and Quadratic Reciprocity Law:** Quadratic residues- Legendre's symbol and its properties- Evaluation of  $\left(\frac{-1}{p}\right)$  and  $\left(\frac{2}{p}\right)$  Gauss lemma-The quadratic reciprocity law - Applications of the reciprocity law - The Jacobi symbol- Applications to Diophantine equations.

(Text 1: Sections 9.1 to 9.8)

**Primitive Roots:** The exponent of number mod  $m$  and primitive roots- Primitive roots and reduced residue; system- The nonexistence of primitive roots mod  $2a$  for  $a \geq 3$ - The existence of primitive roots mod  $p$  for odd primes  $p$ - Primitive roots and quadratic residues - The existence of primitive roots and  $p^a$ - The existence of primitive roots mod  $2 p^a$  -The nonexistence of Primitive roots in the remaining cases- The number of primitive roots mod  $m$ .

(Text 1: Sections 10.1 to 10.9)

#### Unit III

**Algebraic Backgrounds:** Symmetric polynomials- modules- free abelian groups  
(Text 2, Section 1.4-1.6)

**Algebraic Numbers:** Algebraic numbers- Conjugates and Discriminants- Algebraic integers  
(Text 2: Section 2.1 to 2.3)

**Unit IV****Algebraic Numbers:** Integral bases- Norms and Traces- Rings of integers.

(Text 2: Section 2.4 to 2.6)

**Quadratic and Cyclotomic fields:** Quadratic fields-Cyclotomic fields.

(Text 2: Sections 3.1 to 3.2)

**Text Book:**

1. Tom M Apostol, *Introduction to Analytic Number Theory*, Springer, 1976
2. Ian Stewart and David Tall, *Algebraic Number Theory and Fermat's last theorem* (Third Edition), A K Peters/CRC Press, 2001

**References**

1. D. M. Burton, *Elementary Number Theory* (Sixth Edition), McGraw Hill, 2005
2. G. H. Hardy and E. M. Wright, *Introduction to the theory of numbers*, Oxford International Edn, 1985
3. Hurwitz & N. Kritiko, *Lectures on Number Theory*, Springer Verlag, 1986
4. T. Koshy, *Elementary Number Theory with Applications*; Harcourt / Academic Press, 2002
5. D. Redmond, *Number Theory: Monographs & Texts in Mathematics No: 220*, Marcel Dekker Inc., 1994
6. P. Ribenboim, *The little book of Big Primes*, Springer Verlag, New York, 1991
7. K.H. Rosen, *Elementary Number Theory and its applications* (Third Edn.), Addison Wesley PubCo., 1993
8. P. Samuel, *Theory of Algebraic Numbers*, Herman Paris Houghton Mi in, NY, 1975
9. S. Lang, *Algebraic Number Theory*, Addison Wesley Pub Co., Reading, Mass, 1970)
10. Z.I. Borevich & I.R. Shafarevich, *Number Theory*, Academic Press, NY, 1966. Esmonde and Ram Murthy, *Problems in Algebraic Number Theory*, Springer Verlag, 2000.

**MSMAT03E02: Calculus of Variations****Course Objective:** To enable the students to acquire basic knowledge in Calculus of Variations**Course Learning outcome:** After successful completion of the course, student will be able to understand the basic theory of calculus of variations, get acquainted with Euler equations and apply them in solving extremal problems.**Unit I****Elements of the Theory:** Functionals- Some simple variational problems, Function spaces, The variation of a functional- A necessary condition for an extremum, The simplest variational problem- Euler's equation, The case of several variables, A simple variable end point problem, The variational Derivative, Invariance of Euler's equation,**Further Generalizations:** The fixed end point problem for  $n$  unknown functions (Chapter 1; Chapter 2: Section 9)

**Unit II**

**Further Generalizations:** Variational problems in parametric form, Functional Depending on higher order derivatives, Variational Problems with subsidiary conditions

**General Variations of a Functional:** Derivation of the basic formula, End points lying on two given curves or surfaces,

(Chapter 2: Sections 10 to 12; Chapter 3: Sections 13 to 14)

**Unit III**

**General Variations of a Functional:** Broken extremals –The Weierstrass-Erdmann conditions

**The Canonical Form of the Euler Equations and related topics:** The canonical form of the Euler equations, First Integrals of the Euler equations, The Legendre transformation, Canonical transformations, Noether's Theorem, The principle of least action, Conservation laws, The Hamilton-Jacobi Equation- Jacobi's Theorem

(Chapter 3: Section 15; Chapter 4)

**Unit IV**

**The Second Variation- Sufficient condition for a Weak Extremum:** Quadratic functionals- The second variation of a functional, The formula for the second variation- Legendre's

condition, Analysis of the quadratic functional  $\int_a^b (Ph'^2 + Qh^2) dx$ , Jacobi's necessary condition-

More on conjugate points, Sufficient conditions for a weak extremum

(Chapter 5: Sections 24 to 28)

**Text Book:** I M. Gelfand and S.V Fomin, *Calculus of Variations*, Prentice Hall Inc, N.Y, 1963

**References:**

1. G. A. Bliss, *Calculus of Variations*, Open Court Publishing Co. Chicago, 1925
2. O. Bolza, *Lecture on Calculus of Variations*, G.E Stinchar & Co. NY, 1931
3. R. Courant and D. Hilbert, *Methods of Mathematical Physics* (Vol. 1), Wiley Eastern, 1975
4. I. Elsgoltz, *Differential Equations and Calculus of Variations*, Mr Publishers Moscow, 1973
5. M. Morse, *The Calculus of Variations*, American Mathematical Society, 1934

**MSMAT03E03: Algebraic Topology**

**Course Objective:** To enable the students to acquire advanced level knowledge in Algebraic Topology.

**Course Learning Outcome:** After successful completion of the course, student will be able to understand the basics of algebraic topology and understand the fundamental group from a different perspective.

**Unit I**

Geometric Complexes and Polyhedra: Introduction, Examples, Geometric Complexes and Polyhedra, Orientation of Geometric Complexes.

Chapter 1: Sections 1.1 to 1.4)

**Unit II**

Simplicial Homology Groups: Chains, Cycles, Boundaries and Homology Groups, Examples of Homology Groups, The Structure of Homology Groups, The Euler-Poincare Theorem, Pseudomanifolds, and Homology Groups of  $S^n$ .

(Chapter 2: Sections 2.1 to 2.5)

**Unit III**

Simplicial Approximation: Introduction, Simplicial Approximation, Induced Homomorphisms on the Homology Groups, The Brouwer Fixed Point Theorem and Related Results.

(Chapter 3: Sections 3.1 to 3.4)

**Unit IV**

The Fundamental Group: Introduction, Homotopic Paths and The Fundamental Group, The Covering Homotopy property for  $S^1$ , Examples of Fundamental Groups.

(Chapter 4: Sections 4.1 to 4.4)

**Text Book:** Fred H. Croom, *Basic Concepts of Algebraic Topology*, Springer, 1978

**References**

1. S. Eilenberg and N. Steenrod, *Foundations of Algebraic Topology*, Princeton Univ. Press; 1952
2. S. T. Hu, *Homology Theory*, Holden-Day, San Francisco, 1966
3. C. T. C. Wall, *A Geometric Introduction to Topology*, Addison-Wesley Pub. Co. Reading Mass, 1972
4. W. S. Massey, *Algebraic Topology: An Introduction*, Springer Verlag N Y, 1977

**MSMAT03E04: Numerical Analysis and Computing**

**Course Objective:** The aim is to provide an introduction to the fundamental concepts of numerical analysis and computing.

**Course Learning Outcome:** After successful completion of the course, student will be able to understand different methods of finding numerical solutions of a system of equations.

**Unit I**

**Principles of Numerical Calculations :** Common Ideas and Concepts, Fixed-Point Iteration, Newton's Method, Linearization and Extrapolation, Finite Difference Approximations, Some Numerical Algorithms, Solving a Quadratic Equation, Recurrence Relations. Divide and Conquer Strategy. Matrix Computations, Matrix Multiplication, Solving Linear Systems by LU Factorization, Sparse Matrices and Iterative Methods, Software for Matrix Computations. The

Linear Least Squares Problem, Basic Concepts in Probability and Statistics, Characterization of Least Squares Solutions, The Singular Value Decomposition, The Numerical Rank of a Matrix. Numerical Solution of Differential Equations, Euler's Method, Introductory Example, Second Order Accurate Methods.

(Chapter 1: Sections 1.1 to 1.5 except 1.5.4)

## Unit II

**How to Obtain and Estimate Accuracy**, Basic Concepts in Error Estimation, Sources of Error, Absolute and Relative Errors, Rounding and Chopping, Computer Number Systems, The Position System, Fixed- and Floating-Point Representation, IEEE Floating-Point Standard, Elementary Functions, Multiple Precision, Accuracy and Rounding Errors, Floating-Point Arithmetic, Basic Rounding Error Results, Statistical Models for Rounding Errors, Avoiding Overflow and Cancellation.

(Chapter 2: Sections 2.1 to 2.3)

## Unit III

**Interpolation and Approximation:** The Interpolation Problem, Bases for Polynomial Interpolation, Conditioning of Polynomial, Interpolation Formulas and Algorithms, Newton's Interpolation, Inverse Interpolation, Barycentric Lagrange Interpolation, Iterative Linear Interpolation, Fast Algorithms for Vandermonde Systems, The Runge Phenomenon, Generalizations and Applications, Hermite Interpolation, Complex Analysis in Polynomial Interpolation, Rational Interpolation, Multidimensional Interpolation, Piecewise Polynomial Interpolation, Bernštein Polynomials and Bézier Curves, Spline Functions, The B-Spline Basis, Least Squares Splines Approximation. The Fast Fourier Transform. The FFT Algorithm.

(Chapter 4: Sections 4.1 to 4.4 and 4.7.1)

## Unit IV

**Numerical Integration:** Interpolatory Quadrature Rules, Treating Singularities, Classical Formulas, Super-convergence of the Trapezoidal Rule, Higher-Order Newton-Cotes' Formulas, Integration by Extrapolation, The Euler-Maclaurin Formula, Romberg's Method, Oscillating Integrands Adaptive Quadrature

(Chapter 5: Sections 5.1 except 5.1.6,5.2)

**Text Book:** Germund Dahlquist, Ake Björck, *Numerical Analysis in Scientific Computing* (Vol.1), Cambridge University press, 2008

## References:

1. William H. Press, *Numerical Recipes in C – The art of scientific computing* (Third edition), Cambridge University Press, 1992

2. Laurene V. Fausett, *Applied Numerical Analysis using MATLAB*. ( Second edition), Pearson, 2007
3. David Kincaid, Chency et.al., *Numerical Analysis: Mathematics of Scientific Computing*,(Third edition), Cengage Learning ( Pub), 2012
4. Deuffhard P. & A. Hofmann,*Numerical Analysis in Modern Scientific Computing*, Springer, 2002.

## Open Elective Papers (Third Semester)

### MSMAT03001 : Graph Theory

**Course Objective:** To enable the students to acquire advanced level knowledge in Graph Theory

**Course Learning outcome:** After successful completion of the course, student will develop knowledge n connectivity in graphs, Independent sets and Matchings, Edge and vertex colourings and related concepts.

#### Unit I

**Graphs and Subgraphs:** Graphs and Simple graphs, Graph Isomorphism, Incidence and adjacency matrix, Subgraphs, Vertex degrees, Paths and Connection, Cycles.

**Trees:** Trees, Cut Edges and Bonds, Cut Vertices.

(Chapter 1: Sections 1.1 to 1.7; Chapter 2: Sections 2.1 to 2.3)

#### Unit II

**Connectivity:** Connectivity, blocks.

**Euler Tours and Hamilton Cycles:** Euler Tours, Hamilton Cycles(up to (including) Lemma 4.4.1)

**Matchings:** Matchings, Matchings and Coverings in bipartite Graphs.

(Chapter 3: Section 3.1 and 3.2; Chapter 4: Sections 4.1 and 4.2 ( Excluding Lemma 4.4.2 to Corollary 4.6 and related Exercises);Chapter 5: Sections 5.1 and 5.2)

#### Unit III

**Edge Colourings:** Edge Chromatic Number, Vizing's Theorem.

**Vertex Colorings:** Chromatic Number, Book's Theorem, Hajo's Conjecture.

(Chapter 6: Sections 6.1 and 6.2; Chapter 8: Sections 8.1 to 8.3)

#### Unit IV

**Independent Sets and Cliques:** Independent Sets, Ramsey's Theorem.

**Planar Graphs:** Plane and Planar Graphs, Dual Graphs, Euler's Formula, Bridges

(Chapter 7: Section 7.1 and 7.2; Chapter 9: Section 9.1 to 9.4)

Text Book: J.A.Bondy and U.S.Murty, *Graph Theory with Applications*, The MacMillan Press Ltd, 1976

**References:**

1. F. Harary, *Graph Theory*, Narosa Publishing House, 2001
2. Narasimha Deo, *Graph Theory with applications to Engineering and Computer Science*, PHI, 1979
3. O. Ore, *Graph and Their uses*, Random House Inc, NY, 1963
4. K. D. Joshi, *Foundations of Discrete Mathematics*, Wiley Eastern Ltd, 1989

**MSMAT03002: Discrete Mathematics**

**Course Objective:** To enable the students to acquire advanced level knowledge in Discrete Mathematics

**Course Learning outcome:** After successful completion of the course, student will develop knowledge in Combinatorics and Graph theory

**Unit I**

General Counting Methods for Arrangement and Selections, Generating Functions

( Chapter 5: Sections 5.1 to 5.5; Chapter 6: Sections 6.1 to 6.5)

**Unit II**

Recurrence Relations and Inclusion and Exclusion

( Chapter 7: Sections 7.1 to 7.5; Chapter 8: Sections 8.1 to 8.3)

**Unit III**

Polya's Enumeration Formula

(Chapter 9: Sections 9.1 to 9.4)

**Unit IV**

**Quick Review of Basic Concepts in Graph Theory as in Chapter 1 of the text**

**(Questions from this chapter need not be included for the external examination)**

Search Trees and Spanning Trees, The Travelling Salesperson Problem, Tree Analysis of Salesperson Problems, Tree Analysis of Sorting Algorithms

(Chapter 3: Sections 3.2 to 3.4)

**Text:** Alan Tucker, *Advanced Combinatorics* (Sixth Edition), John Wiley and Sons Inc., 2012

**References:**

1. R. Brualdi, *Introductory Combinatorics*, Prentice Hall Inc., 5<sup>th</sup> Edition, 2009.
2. D. Mazur, *Combinatorics: A guided tour*, Mathematical Association of America, 2009.
3. R. Merris, *Combinatorics* (Second edition) John Wiley and Sons, 2003.
4. W. Wallis and J. George, *Introduction to Combinatorics*, CRC Press, 2011.

**MSMAT03003: Operations Research**

**Course Objective :** Identify and develop the mathematical tools that are needed to solve optimization problems.

**Course Learning outcome:** After successful completion of the course, student will be able to understand different techniques involved in operations research.

**Unit I**

Linear programming in two-dimensional spaces. General LP problem. Feasible, basic and optimal solutions, simplex method, simplex tableau, finding the first basic feasible solution, degeneracy, simplex multipliers. (Chapter 3: Sections 1 to 15).

**Unit II**

The revised simplex method. Duality in LP problems, Duality theorems, Applications of duality, Dual simplex method, summary of simplex methods, Applications of LP. (Chapter 3: Sections 16 to 22)

**Unit III**

Transportation and Assignment problems  
(Chapter 4)

**Unit IV**

Integer programming. Theory of games  
(Chapters 6 and 12)

**Text Book:** K. V. Mital and C. Mohan, *Optimisation Methods in Operations Research and Systems Analysis* (Third Edition), New Age International, 1996

**References:**

1. Wagner, *Operations Research*, Prentice Hall India
2. A. Ravindran, Don T. Philips, James Solberg, *Operations Research, Principles and Practice* (Third Edition), John Wiley
3. G. Hadley, *Linear Programming*, Addison Wesley
4. KantiSwarup, P.K.gupta, Man Mohan, *Operations Research*, S. Chand & Co., 2007
5. Premkumar Gupta and D. S. Hira, *Operations Research*, S. Chand & Company Ltd., 1995.

## **MSMAT03004: Fuzzy Mathematics**

**Course Objective:** The aim is to provide an introduction to the fundamental concepts of Fuzzy Mathematics.

**Course Learning outcome:** After successful completion of the course, student will be able to understand the basics of fuzzy mathematics.

### **Unit I**

From classical (crisp) sets to fuzzy sets: characteristics and significance of the paradigm shift. Additional properties of  $\alpha$ -cuts. Representation of fuzzy sets. Extension principle for fuzzy sets.  
(Chapter 1 & Chapter 2)

### **Unit II**

Operations on fuzzy sets. Types of operations. Fuzzy complements. t-norms, t-conorms. Combinations of operations. Aggregate operations, Fuzzy numbers Arithmetic operations on intervals. Arithmetic operations on fuzzy numbers. Lattice of fuzzy numbers  
(Chapter 3: Sections 3.1 to 3.4; Chapter 4: Sections 4.1 to 4.5)

### **Unit III**

Crisp and fuzzy relations, projections and cylindric extensions, binary fuzzy relations, binary relations on a single set, Fuzzy equivalence relations, Compatibility and ordering relations.  
(Chapter 5: Sections 5.1 to 5.6)

### **Unit IV**

Fuzzy morphisms. sup-i, inf-  $\omega$  compositions of fuzzy relations. Fuzzy logic. Fuzzy propositions. Fuzzy quantifiers. Linguistic hedges. Inference from conditional, conditional and qualified and quantified propositions  
(Chapter 5: Sections 5.8 to 5.10 ; Chapter 8)

**Text Book:** G. J. Klir and Bo Yuan, *Fuzzy sets and Fuzzy logic Theory and Applications*, PHI, 1995

### **References:**

1. H. J. Zimmermann, *Fuzzy Set Theory and its Applications*, Kluwer, 1985
2. H. J. Zimmermann, *Fuzzy Sets, Decision Making and Expert Systems*, Kluwer, 1987
3. D. Dubois & H. Prade, *Fuzzy Sets and Systems: Theory and Applications*, Academic Press, 1980

## MSMAT03005: Coding Theory

**Course Objective:** The aim is to introduce the fundamental concepts of coding theory.

**Course Learning outcome:** After successful completion of the course, student will be able to understand the basics of coding theory.

### Unit I

Introduction to Coding Theory. Correcting and detecting error patterns. Weight and distance. MLD and its reliability. Error-detecting codes. Error correcting codes. Linear codes (Chapter 1; Chapter 2: Sections 2.1 to 2.5)

### Unit II

Generating matrices and encoding. Parity check matrices. Equivalent codes. MLD for linear codes. Reliability of IMLD for linear codes, Some bounds for codes, Perfect codes, Hamming codes. Extended codes extended Golay code and Decoding of extended Golay code (Chapter 2: Sections 2.6 to 2.12; Chapter 3: Sections 3.1 to 3.6)

### Unit III

The Golay code, Reed-Muller codes, Fast decoding of  $RM(1,m)$ , Cyclic linear codes. Generating and parity check matrices for cyclic codes. Finding cyclic codes. Dual cyclic codes (Chapter 3: Sections 3.7 to 3.9; Chapter 4)

### Unit IV

BCH codes. Decoding 2-error-correcting BCH code. Reed-Solomon codes. Decoding (Chapter 5; Chapter 6: Sections 6.1 to 6.3)

**Text Book :** – D.R. Hankerson, D. G. Hoffman, D. A. Leonard, C. C. Lindner, K. T. Phelps, C. A. Rodger and J. R. Wall, *Coding Theory and Cryptography The Essentials* (Second edition), Marcel Dekker (2000)

### References:

1. J. H. van Lint, *Introduction to Coding Theory*, Springer Verlag, 1982
2. E. R. Berlekamp, *Algebraic Coding Theory*, McGraw Hill, 1968

## **MSMAT03006: Automata and Formal Languages**

**Course Objective:** To enable the students to acquire basic knowledge in Automata Theory

**Course Learning outcome:** After successful completion of the course, student will be able to understand the basic theory of Automata and Formal languages.

### **Unit I**

Strings, Alphabets and Languages, Finite state systems, Basic definitions, Nondeterministic finite automata, Finite automata with  $\epsilon$  moves.

(Chapter 1:Section 1.1; Chapter 2: Sections 2.1 to 2.5)

### **Unit II**

Regular expressions, Finite automata and regular expressions, Finite automata Applications of regular expressions, The pumping lemma for regular languages, Closure properties of regular sets, Decision properties for regular sets, Minimizing finite automata.

(Chapter 3: Sections 3.1 to 3.3; Chapter 4: Sections 4.1 to 4.4)

### **Unit III**

Motivation and introduction to CFG, Context-free grammars, Derivation trees, Ambiguity in context-free grammars, Informal description, Definitions of Pushdown Automata and Context free languages, The language of a PDA, Equivalence of PDA's and CFG's

(Chapter 5: Sections 5.1, 5.2, 5.4; Chapter 6: Sections 6.1 to 6.3) (Proof of Theorem 6.9 excluded)

### **Unit IV**

Context free languages, pumping lemma for context free languages, Closure properties of context free languages, Introduction to Turing machine, The Turing Machine model, Programming techniques for Turing Machines

(Chapter 7: Sections 7.1 to 7.3; Chapter 8: Sections 8.1 to 8.3)

**Text Book:** John E. Hopcroft, Rajeev Motwani and Jeffery D. Ullman, *Introduction to Automata Theory, Languages and Computation* (Second Edition), Pearson, 2001.

### **References:**

1. G.E.Revesz, *Introduction to Formal Languages*, Dover, 2012.
2. K. L. P. Mishra, N. Chandrasekharan, *Theory of Computer Science: Automata Languages and Computation*, PHI, 2006.
3. M. Sipser, *Introduction to the Theory of Computation*, (Third edition), Cengage India Private Limited, 2014.
4. P.Linz, *An Introduction to Formal Languages and Automata* (Sixth Edition), Jones and Bartlett student, 2012.

## SEMESTER-IV

### MSMAT04C14: Operator Theory

**Course Objective:** To enable the students to advance from functional analysis to operator theory

**Course Learning outcome:** After successful completion of the course, student will be able to understand the advanced level operator theory and their interplay with other branches of higher mathematics.

#### Unit I

**Fundamental Theorems for Normed and Banach Spaces:** Adjoint operator, Reflexive spaces (Definitions and statement of Results from section 4.6), Definition of Category, Baire's Category Theorem (Statement only proof excluded), Uniform Boundedness Theorem. Definition of Strong convergence. Definition of Weak convergence. Open Mapping Theorem, Closed Linear Operators, Closed Graph Theorem.

(Chapter 4: Sections 4.5, 4.6(Definitions and statement of Results only), 4.7-1, 4.7-2 (Statement only), 4.7-3, 4.8-1, , 4.8-2, 4.12 and 4.13).

#### Unit II

**Approximation Theory:** Approximation in Normed Spaces, Uniqueness, Strict Convexity, Approximation in Hilbert Space.

**Spectral Theory in Normed Spaces:** Basic Concepts, Spectral Properties of Bounded Linear Operators. Further Properties of Resolvent and Spectrum, Use of Complex Analysis in Spectral Theory.

(Chapter 6: Sections 6.1, 6.2 and 6.5; Chapter 7: Sections 7.2 to 7.5)

#### Unit III

**Compact Operators and Their Spectrum:** Compact Linear Operators on Normed Spaces, Further Properties of Compact Linear Operators. Spectral Properties of Compact Linear Operators on Normed Spaces, Further Spectral Properties of Compact Linear Operators.

(Chapter 8: Sections 8.1 to 8.4)

#### Unit IV

**Spectral Theory of Bounded Self-Adjoint Linear Operators:** Spectral Properties of Bounded Self-Adjoint Linear Operators, Further Spectral Properties of Bounded Self-Adjoint Linear Operators, Positive Operators, Square Roots of a Positive Operator (Statement only proof of the theorem is excluded), Projection Operators, Further Properties of Projections.

(Chapter 9: Sections 9.1 to 9.6 (All proofs in section 9.4 are omitted)).

**Text Text Book:** Erwin Kreyszig, *Introductory Functional Analysis with Applications*, John Wiley, 1991

#### References:

1. Balmohan V Limaye, *Functional Analysis* (Second Edition), New Age International Publishers, 1996
2. M.T Nair, *Functional Analysis: A First Course*; Prentice Hall of India, 2021
3. Walter Rudin, *Functional Analysis*, McGraw-Hill Inc., US, 1978

4. B. Choudhary, Sudarsan Nanda, *Functional Analysis with Applications* (Second edition), New Age International (P) Ltd., 2003
5. Walter Rudin, *Real and Complex Analysis*, McGraw Hill Education, 2017
6. J.B Conway, *A Course in Functional Analysis*, Springer, 2010

### **MSMAT04C15: Complex Function Theory**

**Course Objective:** To enable the students to acquire advanced level knowledge in complex function theory

**Course Learning outcome:** After successful completion of the course, student will develop knowledge in advanced complex analysis and would be capable to apply these knowledge in solving Harmonic PDEs.

#### **Unit I**

Weierstrass Factorization Theorem, Factorization of the sine function, The gamma function (Chapter 7: Sections 4 to 7)

#### **Unit II**

The Riemann zeta function, Runge's Theorem, Simple connectedness (Chapter 7: Section 8; Chapter 8: Sections 1 and 2))

#### **Unit III**

Mittag-Leffler's Theorem, Schwarz Reflection Principle, Analytic continuation along a path (Chapter 8: Section 3; Chapter 9: Sections 1 and 2)

#### **Unit IV**

Mondromy Theorem, Basic Properties of Harmonic functions, Harmonic functions on a disk (Chapter 9: Section 3; Chapter 10: Sections 1 and 2))

**Text Book:** John B Conway, *Functions of One Complex Variable* (Second Edition), Springer, 1995

#### **References:**

1. Louis Pennise, *Elements of Complex Variable*, Half, Richart & Winston, 1976
2. H. Silverman, *Complex Variable*, Houghton Mifflin Complex, Boston, 1975.
3. Walter Rudin, *Real and Complex Analysis* (Third Edition) McGraw Hill International Edition, 1967
4. E. T Copson, *An Introduction to the Theory of a Complex Variables*, Oxford University Press, 1974.
5. Lars V. Ahlfors, *Complex Analysis* (Third Edition), McGraw Hill Education, 2017
6. Theodore W. Gamelin, *Complex Analysis*, Springer, 2001

## **MSMAT04C16: Project/Dissertation**

**Course Objective:** Enable students to learn independently and orient them towards research in Mathematics.

**Course Learning Outcome:** After successful completion of the project work, student will be able to study or research in a topic that is beyond the regular classroom learning in both rigor and content. Further, students will be able to produce reports that exhibit both the background and the conclusions reached as a result of such study or research.

### **Guidelines for project:**

1. The project work can be done in any advanced level topic of Mathematics.
2. The student should not have studied this topic in the regular class.
3. Each student must be assigned a supervisor from the Department for carrying out the project work.
4. Students may present the progress of the project work to the supervisor regularly, which shall be considered for the continuous evaluation of the project by the supervisor
5. The project report must be self-contained as far as possible
6. The students must submit a report at the end of the project, which is duly signed by the supervisor on or before the date stipulated by the Department.

### **Guidelines for the preparation of dissertation on project:**

1. Arrangement of contents shall be as follows:
  1. Cover page and title page
  2. Bonafide certificate
  3. Declaration by the student
  4. Acknowledgement
  5. Table of contents
  6. List of tables
  7. List of figures
  8. List of symbols, Abbreviations and Nomenclature
  9. Chapters
  10. Appendices
  11. References

2. Page dimension and typing instruction:

The dimension of the dissertation on project should be in A4 size. The dissertation should be typed in bond paper and bound using flexible cover of the thick white art paper or spiral binding. The general text shall be typed preferably in Latex with font size 12. The content of the report shall be around 40 pages.

3. Bonafide certificate shall be in the following format:

**CERTIFICATE**

This is to certify that the project entitled .....(title ) submitted to the Kannur University in partial fulfilment of the requirements of Post Graduate Degree in .....(subject), is a bonafide record of studies and work carried out by .....(Name of the student) under my supervision and guidance.

Office seal      Signature, name, designation and official address of the Supervisor.

Date

4. Declaration by the student shall be in the following format:

**DECLARATION**

I.....( Name of the candidate) hereby declare that this project titled .....( title) is a bonafide record of studies and work carried out by me under the supervision of .....( Name, designation and official address of the supervisor), and that no part of this project, except the materials gathered from scholarly writings, has been presented earlier for the award of any degree or diploma or other similar title or recognition.

Date:

Signature and name of the student

## **MSMAT04C17: Comprehensive Viva - Voce**

**Course Objective:** The objective of comprehensive viva-voce is to ensure the overall knowledge and effective communication of each student in various topics of the syllabus.

**Course Learning Outcome:** After the successful completion of the comprehensive viva-voce, students will have an overall idea about the various topics covered in the course. Students will also be able to face interviews with confidence.

### **Guidelines for comprehensive viva-voce:**

1. Questions can be from any topic studied in the entire course with more emphasis on core papers.
2. Students may be asked to present any topic in the programme decided by the Examiners.

## Core Elective Papers (Fourth Semester)

### MSMAT04E05: Commutative Algebra

**Course Objective:** To enable the students to acquire basic knowledge in commutative algebra

**Course Learning outcome:** After successful completion of the course, student will be able to understand the basic theory of commutative algebra and get acquainted with ideals, modules and decomposition theorems.

#### Unit I

**Rings and Ideals:** Rings and Ring Homomorphism, Ideals, Quotient Rings, Zero Divisors, Nilpotent Elements, Unit, Prime Ideals and Maximal Ideals, Nilradical and Jacobson Radical, Operations on Ideals, Extension and Contraction  
(Chapter 1)

#### Unit II

**Modules:** Modules and Module Homomorphism, Submodules and Quotient Modules, Operations on Sub modules, Direct Sum and Product, Finitely Generated Modules, Exact Sequences.

**Rings and Modules of Fractions:** Rings and Modules of Fractions, Local Properties,  
(Chapter 2 (relevant sections); Chapter 3(relevant sections))

#### Unit III

**Rings and Modules of Fractions:** Extended and Contracted Ideals

**Primary Decomposition:** Primary Decomposition.  
(Chapter 4; Chapter 5 (relevant sections))

#### Unit IV

**Chain Conditions:** Chain conditions

**Noetherian Rings:** Primary decomposition in Noetherian Rings  
(Chapter 6; Chapter 7 (Omit Proposition 7.8, Proposition 7.9 and Corollary 7.10))

**Text Book:** M. F. Atiyah and I. G. Macdonald, *Introduction to commutative Algebra*, Addison Wiley, 1969

#### References:

1. N. Bourbaki, *Commutative Algebra*, Paris Herman, 1961
2. D. Burton, *A first course introduction to Rings and Ideals*, Wesley, 1970
3. N. S. Gopalakrishnan, *Commutative Algebra*, Oxonian Press, 1984
4. T. W. Hungerford, *Algebra*, Springer Verlag, 1974
5. D. G. Northcott, *Ideal Theory*, Cambridge University Press, 1953
6. O. Zariski and P. Samuel, *Commutative Algebra* (Vol I and II). Van Nostrand, Princeton, 1960

## MSMAT04E06: Fourier and Wavelet Analysis

**Course Objective:** To enable the students to acquire basic theory of Fourier and wavelet analysis

**Course Learning outcome:** After successful completion of the course, student will be able to understand the basic theory of wavelet analysis in  $Z_n$ ,  $Z$  and  $R$ .

### Unit I

**Wavelets on  $Z_n$ :** Construction of Wavelets on  $Z_n$ - The First Stage, Construction of Wavelets on  $Z_n$ - The Iteration Step.

(Chapter 3: Sections 3.1 and 3.2)

### Unit II

**Wavelets on  $Z_n$ :** The Haar System, the Shannon wavelets and the Daubechies's D6 wavelets on  $Z_n$ .

**Wavelets on  $Z$ :**  $l^2(Z)$ , Complete Orthonormal sets in Hilbert Spaces,  $L^2([-\pi, \pi])$  and Fourier Series,

(Chapter 3: Examples 3.32, 3.33 and 3.35 of Section 3.3.; Chapter 4: Sections 4.1 to 4.3)

### Unit III

**Wavelets on  $Z$ :** The Fourier transforms and convolution on  $l^2(Z)$ , First Stage Wavelets on  $Z$ , The Iteration Step for Wavelets on  $Z$ .

(Chapter 4: Sections 4.4 to 4.6)

### Unit IV

**Wavelets on  $R$ :**  $L^2(R)$  and Approximate Identities, The Fourier Transform on  $R$ .

(Chapter 5: Section 5.1 to 5.2)

**Text Book:** M.W. Frazier, *An Introduction to Wavelets through Linear Algebra*, Springer, 1999

### References:

1. G. Bachman, L. Narici, E. Beckenstein, *Fourier and Wavelet Analysis*, Springer, 2000
2. I. Daubechies, *Ten Lectures on Wavelets*, SIAM, 1992
3. C. Heil, *A Basis Theory Primer*, Birkhauser, 2011
4. D. F. Walnut, *An Introduction to Wavelet Analysis*, Birkhauser, 2002

## MSMAT04E07: Probability Theory

**Course Objective:** To enable the students to acquire advanced level knowledge in probability theory.

**Course Learning Outcome:** After successful completion of the course, student will develop knowledge in defining the probability space of a random variable, the distribution function and convergence of distribution function of a random variable.

**Unit I**

Sets and Classes of Events, Random Variables

(Chapter 1: sections 1.1 to 1.4; Chapter 2: Sections 2.1 to 2.3)

**Unit II**

Probability Spaces, Distribution Functions.

(Chapter 3: Sections 3.1 to 3.5; Chapter 4: Sections 4.1 to 4.4)

**Unit III**

Expectation and Moments, Convergence of Random Variables.

(Chapter 5: Section 5.1 to 5.3; Chapter 6: Sections 6.1 to 6.6)

**Unit IV**

Characteristic Functions, Convergence of Distribution Functions.

(Chapter 7: Section 7.1 to 7.5; Chapter 8: Sections 8.1 to 8.3)

**Text Book:** B.R Bhat, *Modern Probability Theory* (Second Edition), New Age International Pvt Ltd, New Delhi.

**References:**

1. A.K. Basu, *Measure and Probability*, Prentice Hall of India, 2003.
2. A.N Kolmogorov, *Foundations of Probability*, Chelsea, NY, 1950
3. M. Loeve, *Probability Theory*, Van-Nostrand, Princeton, 1963
4. Y. S. Chows and H. Tiecher, *Probability Theory*, Springer Verlag, 1988
5. P. Billingsley, *Probability and Measure*, John Wiley & Sons, NY(1979)

**MSMAT04E08: Advanced Differential Geometry**

**Course Objective:** The aim is to introduce the advanced level topics of differential geometry

**Course Learning Outcome:** After successful completion of the course, student will be able to understand and apply the theory of manifolds and Lie algebra

**Unit I**

Differential Manifolds, Smooth maps and diffeomorphisms,, Tangent spaces to a manifold, Derivatives of smooth maps, Immersions and submersions, Submanifolds.

(Chapter 2: Sections 2.1 to 2.6)

**Unit II**

Vector fields, Flows and exponential map, Frobenius theorem, Lie groups and Lie algebras, Homogenous spaces

(Chapter 2: Sections 2.7 to 2.11)

**Unit III**

Tensor Analysis, Multilinear algebra, Exterior algebra, Tensor fields, The exterior derivative, Lie derivatives

(Chapter 3: Sections 3.1 to 3.5)

**Unit IV**

Integration, Orientable manifolds, Integration on manifolds, Stoke's theorem

(Chapter 4: Sections 4.1 to 4.3)

**Textbook:** S. Kumaresan, *A Course in Differential Geometry and Lie groups*, Hindustan Book Agency, 2002

**References:**

1. John M. Lee, *Introduction to smooth manifolds*, Springer, 2002
2. M. Spivak, *A Comprehensive introduction to Differential Geometry*, Vol 1., Publish or perish 1970
3. Loring W. Tu. , *An Introduction to manifolds*, Springer, 2011
4. J. M. Lee, *Manifolds and Differential Geometry*, American Mathematical Society, 2009
5. J. A. Thorpe, *Elementary Topics in Differential Geometry*, Springer- Verlag, 1979

**Kannur University**  
**Model Question Paper**  
**FIRST SEMESTER M.Sc. DEGREE EXAMINATION**

Mathematics

MSMAT01C01: Abstract Algebra

CBCSS - 2023 Admission Onwards

Time: Three Hours

Maximum : 80 Marks

**Part A**

*Answer any five questions.*

*Each question carries 4marks.*

1. Define the following  
 (i)  $G$  set (ii) Isotropy subgroup of  $x \in X$  where  $X$  is a  $G$  set
2. Define normalizer of a subgroup  $H$  in  $G$  and  $p$ -group.
3. State third isomorphism theorem. Describe with an example.
4. Define ascending central series of the group  $G$  and give an example.
5. Find the product of the polynomials  $f(x) = 4x - 5$  and  $g(x) = 2x^2 - 4x + 2$  in  $\mathbb{Z}_8[x]$
6. Let  $\phi : \mathbb{Z}_{18} \rightarrow \mathbb{Z}_{12}$  be the homomorphism such that  $\phi(1) = 10$ . Find  $\ker \phi$

(5 × 4 = 20 marks)

**Part B**

*Answer any three questions.*

*Each question carries 7 marks*

7. State whether the following statements are true or false. Justify
  - (a) Any abelian group of order 27 is cyclic.
  - (b) Any abelian group of order 14 is cyclic.
  - (c) Any abelian group of order 21 is cyclic.
  - (d) Any abelian group of order 30 is cyclic.
8. Consider a group  $G$ . Let  $Z(G)$  be its centre for  $n \in \mathbb{N}$ , define  $J_n = \{(g_1, g_2, \dots, g_n) \in Z(G) \times Z(G) \times \dots \times Z(G) : g_1 g_2 \dots g_n = e\}$ . As a subset of a direct product groups  $G \times G \times \dots \times G$  ( $n$  times direct product of the group  $G$ ), is
  - (a) not necessarily subgroup

- (b) a subgroup but not necessarily a normal subgroup  
 (c) normal subgroup  
 (d) isomorphic to the direct product  $Z(G) \times Z(G) \times \dots \times Z(G)$  ( $n - 1$  times).
9. Find the isomorphic refinements of the following series
- (a)  $\{0\} < 60\mathbb{Z} < 20\mathbb{Z} < \mathbb{Z}$  and  $\{0\} < 245\mathbb{Z} < 49\mathbb{Z} < \mathbb{Z}$   
 (b)  $\{0\} < \langle 5 \rangle < \mathbb{Z}_{40}$  and  $\{0\} < \langle 4 \rangle < \mathbb{Z}_{40}$
10. Give an example of a ring homomorphism  $\phi : R \rightarrow R'$  where  $R$  has unity 1 and  $\phi(1) \neq 0'$ , but  $\phi(1)$  is not unity of  $R'$
11. Let  $I_1$  be the ideal generated by  $x^2 + 1$  and  $I_2$  be the ideal generated by  $x^3 - x^2 + x + 1$  in  $\mathbb{Q}[x]$ . If  $R_1 = \frac{\mathbb{Q}[x]}{I_1}$  and  $R_2 = \frac{\mathbb{Q}[x]}{I_2}$ , then which of the following options are correct. Give justification for each.
- (a)  $R_1$  and  $R_2$  are fields  
 (b)  $R_1$  is a field but  $R_2$  is not a field  
 (c)  $R_1$  is an integral domain but  $R_2$  is not an integral domain  
 (d)  $R_1$  and  $R_2$  are not integral domains.

(3 × 7 = 21 marks)

### Part C

Answer **any three** questions.

Each question carries **13** marks.

12. (a) State and prove Cauchy's theorem 9  
 (b) Prove that every group of order  $p^2$  is abelian 4
13. (a) Prove that if  $D$  is an integral domain and  $F$  is a field containing  $D$ , then  $F$  contains field of quotients of  $D$ . 9  
 (b) Prove that any two fields of quotients of an integral domain  $D$  are isomorphic. 4
14. (a) State and prove third sylow theorem 9  
 (b) Prove that no group of order 48 is simple 4
15. (a) Let  $G$  be a nonzero free abelian group with a finite basis. Then prove that every basis of  $G$  is finite and all basis have same number of elements. 8  
 (b) Show that a free abelian group contains no nonzero elements of finite order. 5
16. (a) State and prove Eisenstein criterion. 8  
 (b) Prove that a nonzero polynomial  $f(x) \in F[x]$  of degree  $n$  can have at most  $n$  zeros in a field  $F$  5

(3 × 13 = 39 marks)

## Model Question Paper

## FIRST SEMESTER M.Sc. DEGREE EXAMINATION

## Mathematics

## MSMAT01C02: Linear algebra

## CBCSS - 2023 Admission Onwards

Time: Three Hours

Maximum : 80 Marks

## Part A

*Answer any five questions.**Each question carries 4 marks.*

1. Define null space of a linear transformation. Give an example.
2. Let  $T$  be a linear operator on  $R^3$  defined by  $T(x, y, z) = (x + 2y, x + y + z, 2y + 4z)$ . Find the matrix of  $T$  with respect to the standard basis
3. Define annihilator of a subspace  $S$  of a vector space  $V$  over the field  $F$ . Find the annihilator of the zero subspace of a vector space.
4. Show that similar matrices have same characteristic polynomial.
5. (a) Define cyclic vector for a linear operator on a finite-dimensional vector space  $V$ .  
(b) Define  $T$ -annihilator of  $\alpha$
6. True or False. Justify. "Every inner product space is a metric space".

 $(5 \times 4 = 20 \text{ marks})$ 

## Part B

*Answer any three questions.**Each question carries 7 marks*

7. Let  $S : R^3 \rightarrow R^4$  and  $T : R^4 \rightarrow R^3$  be linear transformations such that  $T \circ S$  is identity on  $R^3$ . Then  $S \circ T$  is
  - (a) Identity on  $R^4$
  - (b) One-One but not onto.
  - (c) Onto but not one-one.
  - (d) Neither one- one nor onto.

8. Let  $A$  be a  $2 \times 2$  matrix with complex entries which is non-zero and non diagonal. Pick out the cases when  $A$  is diagonalizable. Justify your answer.
- $A^2 = I$
  - $A^2 = 0$ .
  - All eigen values of  $A$  are equal to 2.

9. Give an  $n \times n$  matrix  $B$ , define  $e^B = \sum_{j=0}^{\infty} \frac{B^j}{j!}$ . Let  $p$  the characteristic polynomial of  $B$ . Then the matrix  $e^{p(B)}$  is.

- $I_{n \times n}$
- $0_{n \times n}$
- $eI_{n \times n}$
- $I_{n \times n}$

Justify your answer.

10. An  $n \times n$  complex matrix satisfies  $A^k = I_n$ , where  $k > 1$  positive integer. Suppose 1 is not an eigen value of  $A$  Then which are true? Justify your answer.

- $A$  is diagonalizable.
- $A^{k-1} + A^{k-2} + \dots + A = 0$
- $\text{tr}(A^{k-1}) + \text{tr}(A^{k-2}) + \dots + \text{tr}(A) = -n$
- $A^{-(k-1)} + A^{-(k-2)} + \dots + A^{-1} = I$

11. Let  $V$  be the inner product space consisting of linear polynomials,  $p : [0, 1] \rightarrow R$  with inner product given by  $(p|q) = \int_0^1 p(x)q(x)dx$ . An orthonormal basis for  $V$  is

- $(1, x)$
- $(1, x\sqrt{3})$
- $(1, (2x - 1)\sqrt{3})$
- $(1, x - 1/2)$

Justify your answer.

(3 × 7 = 21 marks)

### Part C

Answer any three questions.

Each question carries 13 marks.

12. (a) Let  $V$  and  $W$  be vector spaces over the field  $F$  and let  $T$  be a linear transformation from  $V$  into  $W$ . Suppose that  $V$  is finite dimensional, then prove that  $\text{rank}(T) + \text{nullity}(T) = \dim V$ .
- (b) If  $A$  is an  $m \times n$  matrix with entries in the field  $F$ , then prove that  $\text{row rank}(A) = \text{column rank}(A)$

13. (a)  $V$  be a finite-dimensional vector space over the field  $F$ , and let  $W$  be a subspace of  $V$ . Then prove that  $\dim W + \dim W^\circ = \dim V$
- (b) If  $W_1$  and  $W_2$  are subspaces of a finite-dimensional vector space, then prove that  $W_1 = W_2$  if and only if  $W_1^\circ = W_2^\circ$ .
14. (a) State and prove Cayley-Hamilton theorem.
- (b) If  $W$  is an invariant subspace for  $T$ , then prove that  $W$  is invariant under any polynomial in  $T$ .
15. (a) Let  $V$  be a finite dimensional vector space over a field  $F$  and let  $T$  be a linear operator on  $V$ . Then show that  $T$  is triangulable if and only if the minimal polynomial for  $T$  is the product of linear polynomials over  $F$ .
- (b) Let  $T$  be a linear operator on a finite-dimensional space  $V$ . If  $T$  is diagonalizable and if  $c_1, c_2, \dots, c_k$  are the distinct characteristic values of  $T$ , then prove that there exist linear operators  $E_1, E_2, \dots, E_k$  on  $V$  such that (i)  $T = c_1 E_1 + c_2 E_2 + \dots + c_k E_k$   
(ii)  $I = E_1 + E_2 + \dots + E_k$  (iii)  $E_i E_j = 0, i \neq j$
16. State and prove Primary Decomposition Theorem.

(3 × 13 = 39 marks)

**Kannur University**  
**Model Question Paper**  
**FIRST SEMESTER M.Sc. DEGREE EXAMINATION**

Mathematics

MSMAT01C03: Real Analysis

**CBCSS - 2023 Admission Onwards**

**Time: Three Hours**

amp;

amp; **Maximum : 80 Marks**

**Part A**

*Answer any five questions.*

*Each question carries 4 marks.*

1. Define a perfect set. Give an example with justification.
2. Define a compact set. Give an example with justification.
3. Give an example of a continuous function which is not uniformly continuous. Justify.
4. Give an example of a real valued function  $f$  defined on  $[-1, 1]$  such that  $f$  has a local maximum at 0.
5. Define Riemann - Stieltjes integral and what is its connection with Riemann integral.
6. (a) State fundamental theorem of calculus.  
 (b) Define arc in  $\mathbb{R}^k$ .

(5 × 4 = 20 marks)

**Part B**

*Answer any three questions.*

*Each question carries 7 marks*

7. Let  $A$  be the following subset of  $\mathbb{R}^2$  :  $A = \{(x, y) : (x + 1)^2 + y^2 \leq 1\} \cup \{(x, y) : y = x \sin \frac{1}{x}, x > 0\}$ . Then which of the following statements is/are true and which of them is/are false ? justify your answer.
  - (i)  $A$  is connected.
  - (ii)  $A$  is compact.
  - (iii)  $A$  is bounded.

8. Suppose  $f$  is a uniformly continuous mapping of a metric space  $X$  into a metric space  $Y$  and prove that  $\{f(x_n)\}$  is a Cauchy sequence in  $Y$  for every Cauchy sequence  $\{x_n\}$  in  $X$ . Use this result to give an alternate proof of the following fact:  
Let  $E$  be a dense subset of a metric space  $X$ , let  $f$  be a uniformly continuous real function defined on  $E$ . Prove that  $f$  has a continuous extension from  $E$  to  $X$ .
9. Let  $I = [0, 1]$  be the closed unit interval. Suppose  $f$  is a continuous mapping of  $I$  into  $I$ . Prove that  $f(x) = x$  for at least one  $x \in I$ .
10. Suppose  $f$  is differentiable on  $[a, b]$ ,  $f(a) = 0$ , and there is a real number  $A$  such that  $|f'(x)| \leq A|f(x)|$  on  $[a, b]$ . Prove that  $f(x) = 0$  for all  $x \in [a, b]$ .
11. Suppose  $f$  is twice differentiable function on  $(0, \infty)$ ,  $f''$  is bounded on  $(0, \infty)$ , and  $f(x) \rightarrow 0$  as  $x \rightarrow \infty$ . Prove that  $f'(x) \rightarrow 0$  as  $x \rightarrow \infty$ .

(3 × 7 = 21 marks)

### Part C

Answer any three questions.

Each question carries 13 marks

12. (a) Define open sets and closed sets. Prove that a set  $E$  is open if and only if its complement is closed.  
(b) Define limit point of a set. If  $p$  is a limit point of a set  $E$ , then every neighborhood of  $p$  contains infinitely many points of  $E$ .
13. (a) Prove that a mapping  $f$  of a metric space  $X$  into a metric space  $Y$  is continuous on  $X$  if and only if  $f^{-1}(V)$  is open in  $X$  for every open set  $V$  in  $Y$ .  
(b) Prove that if  $f$  is a continuous mapping of a compact metric space  $X$  into a metric space  $Y$ , then  $f(X)$  is compact.
14. (a) Suppose  $f$  is a real differentiable function on  $[a, b]$  and suppose  $f'(a) < \lambda < f'(b)$ . Prove that there is a point  $x \in (a, b)$  such that  $f'(x) = \lambda$ .  
(b) Let  $f$  be defined on  $\mathbb{R}$  by

$$f(x) = \begin{cases} x \sin\left(\frac{1}{x}\right), & \text{if } x \neq 0, \\ 0, & \text{if } x = 0. \end{cases}$$

Prove that  $f$  is a differentiable at all points  $x$  but  $f'$  is not a continuous function.

15. (a) Prove that  $\int_a^b f d\alpha \leq \int_a^b f d\alpha$ .  
(b) If  $f$  is continuous on  $[a, b]$ , then  $f \in \mathcal{R}(\alpha)$  on  $[a, b]$ .
16. (a) If  $a < s < b$ ,  $f$  is bounded on  $[a, b]$ ,  $f$  is continuous at  $s$ , and  $\alpha(x) = I(x - s)$ , then

$$\int_a^b f d\alpha = f(s).$$

(b) Assume  $\alpha$  increases monotonically and  $\alpha' \in \mathcal{R}$  on  $[a, b]$ . Let  $f$  be a bounded real function on  $[a, b]$ . Then  $f \in \mathcal{R}(\alpha)$  if and only if  $f\alpha' \in \mathcal{R}$ . In that case

$$\int_a^b f d\alpha = \int_a^b f(x)\alpha'(x).$$

(3 × 13 = 39 marks)

**Kannur University**

**Model Question Paper**

**FIRST SEMESTER M.Sc. DEGREE EXAMINATION**

**Mathematics**

**MSMAT01C04: Topology**

**CBCSS - 2023 Admission Onwards**

**Time: Three Hours**

**Maximum : 80 Marks**

**Part A**

*Answer any five questions.*

*Each question carries 4 marks.*

1. Let  $X$  be a set and  $\mathcal{T}$  be the collection of all subsets  $U$  of  $X$  such that  $X - U$  is either finite or is all of  $X$ . Show that  $\mathcal{T}$  is a topology on  $X$ .
2. If  $X = \{a, b, c\}$ , let  $\mathcal{T}_1 = \{\phi, X, \{a\}, \{a, b\}\}$  and  $\mathcal{T}_2 = \{\phi, X, \{a\}, \{b, c\}\}$ . Find the smallest topology containing  $\mathcal{T}_1$  and  $\mathcal{T}_2$ , and the largest topology contained in  $\mathcal{T}_1$  and  $\mathcal{T}_2$ .
3. Define order topology.
4. True or false: Every finite point set in a Hausdorff space is closed. Justify
5. Define metrizable space and give an example.
6. Show that every path connected space is connected.

(5 × 4 = 20 marks)

**Part B**

*Answer any three questions.*

*Each question carries 7 marks*

7. Consider the following topologies on  $\mathbb{R}$ .  
 $\mathcal{T}_1$  = the standard topology,  
 $\mathcal{T}_2$  = the  $\mathbb{K}$ -topology  $\mathbb{R}_{\mathbb{K}}$ ,  
 $\mathcal{T}_3$  = the finite complement topology,  
 $\mathcal{T}_4$  = the upper limit topology, having all sets  $(a, b]$  as basis,  
 $\mathcal{T}_5$  = the topology having all sets  $(-\infty, a) = \{x | x < a\}$  as basis.  
 Determine , for each of these topologies , which of the others it contains.

8. Consider the following sets in the dictionary order. Which are linear continua? Explain the reason.
- $\mathbb{Z}_+ \times [0, 1)$
  - $[0, 1) \times \mathbb{Z}_+$
  - $[0, 1) \times [0, 1]$
  - $[0, 1] \times [0, 1)$
9. Which of the following are metrics on  $\mathbb{R}$ . Explain with reasons.
- $d(x, y) = \min\{x, y\}$
  - $d(x, y) = |x - y|$
  - $d(x, y) = |x^2 - y^2|$
  - $d(x, y) = |x^3 - y^3|$
10. Consider the metrics  $d(x, y) = ((x_1 - y_1)^2 + (x_2 - y_2)^2)^{\frac{1}{2}}$  and  $\rho(x, y) = \max\{|x_1 - y_1|, |x_2 - y_2|\}$  on  $\mathbb{R} \times \mathbb{R}$ . Prove or disprove: the topologies induced by  $d$  and  $\rho$  on  $\mathbb{R} \times \mathbb{R}$  are the same.
11. Consider the space  $S = \{(a, b) | a, b \in \mathbb{Q}\} \subset \mathbb{R}^2$ , where  $\mathbb{Q}$  is the set of rational numbers. Then which of the following options are correct. Give justification for each.
- $S$  is connected in  $\mathbb{R}^2$
  - $S^c$  is connected in  $\mathbb{R}^2$ , where  $S^c$  denotes the complement of  $S$
  - $S$  is path connected in  $\mathbb{R}^2$
  - $S^c$  is path connected in  $\mathbb{R}^2$

(3 × 7 = 21 marks)

### Part C

Answer any three questions.

Each question carries 13 marks.

12. (a) Define product topology. If  $\mathcal{B}$  is a basis for the topology of  $X$  and  $\mathcal{C}$  is a basis for the topology of  $Y$ , then show that the collection  $\mathcal{D} = \{B \times C | B \in \mathcal{B} \text{ and } C \in \mathcal{C}\}$  is a basis for the topology of  $X \times Y$ . 7
- (b) Define subbasis for a topological space. Show that the collection  $\mathcal{S} = \{\pi_1^{-1}(U) | U \text{ open in } X\} \cup \{\pi_2^{-1}(V) | V \text{ open in } Y\}$  is a subbasis for the product topology on  $X \times Y$ . 6
13. (a) Let  $X$  be an ordered set in the order topology, let  $Y$  be a subset of  $X$  that is convex in  $X$ . Then show that the order topology on  $Y$  is the same as the topology  $Y$  inherits as a sub space of  $X$ . 7
- (b) Let  $Y$  be a subspace of  $X$ . Then show that a set  $A$  is closed in  $Y$  if and only if it equals the intersection of a closed set of  $X$  with  $Y$ . 6
14. (a) Let  $X$  be a space satisfying  $T_1$  axiom and let  $A$  be a subset of  $X$ . Then show that the point  $x$  is a limit point of  $A$  if and only if every neighborhood of  $x$  contains infinitely many points of  $A$ . 5

- (b) Let  $X$  and  $Y$  be topological spaces and let  $f : X \rightarrow Y$ . Then prove the following statements are equivalent.
- (1)  $f$  is continuous
  - (2) For every subset  $A$  of  $X$ ,  $f(\overline{A}) \subset \overline{f(A)}$
  - (3) For every closed subset  $B$  of  $Y$ , the set  $f^{-1}(B)$  is closed in  $X$ .
  - (4) For each  $x \in X$  and each neighborhood  $V$  of  $f(x)$ , there is a neighborhood  $U$  of  $x$  such that  $f(U) \subset V$  8
15. (a) State and prove Uniform limit theorem. 6
- (b) Let  $p : X \rightarrow Y$  be a quotient map and let  $A$  be a subspace of  $X$  that is saturated with respect to  $p$ . Let  $q : A \rightarrow p(A)$  be the map obtained by restricting  $p$ . Prove the following:
- (a) If  $A$  is either open or closed in  $X$ , then  $q$  is a quotient map.
  - (b) If  $p$  is either an open map or a closed map, then  $q$  is a quotient map. 7
16. (a) Prove that a finite cartesian product of connected spaces is connected. 8
- (b) Prove that a space  $X$  is locally connected if and only if for every open set  $U$  of  $X$ , each component of  $U$  is open in  $X$ . 5

(3 × 13 = 39 marks)

# Kannur University

## Model Question Paper

### FIRST SEMESTER M.Sc. DEGREE EXAMINATION

#### Mathematics

#### MSMAT01C05: Ordinary Differential Equations

#### CBCSS - 2023 Admission Onwards

Time: Three Hours

Maximum : 80 Marks

### Part A

Answer any five questions.

Each question carries 4 marks.

1. Express  $\sin^{-1} x$  in the form of a power series  $\sum a_n x^n$  by solving  $y' = (1 - x^2)^{-1/2}$  in two ways.
2. Write Rodrigues' formula. Find the first four Legendre polynomials.
3. Show that
  - (a)  $\frac{d}{dx} J_0(x) = -J_1(x)$
  - (b)  $\frac{d}{dx} [xJ_1(x)] = xJ_0(x)$
4. State the fundamental existence and uniqueness theorem for a linear system of first order differential equations.
5. Find the exact solution of the initial value problem:  $y' = 2x(1 + y)$ ,  $y(0) = 0$
6. Solve the following initial value problem by Picard's method:

$$\frac{dy}{dx} = z, y(0) = 1$$

$$\frac{dz}{dx} = -y, z(0) = 0$$

(5 × 4 = 20 marks)

### Part B

Answer any three questions.

Each question carries 7 marks

7. Consider the differential equation  $x(1 - x)y'' + [p - (p + 2)x]y' - py = 0$ , where  $p$  is a constant.

- (a) If  $p$  is not an integer, find the general solution near  $x = 0$  in terms of hypergeometric functions.
- (b) Write the general solution found in (a) in terms of elementary functions.
- (c) What happens when  $p = 1$ ? Find the general solution in this case.
8. Show that the change of dependent variable defined by  $y = t''w$  transforms  $y'' + \left[ \frac{(1-a-b) - (2-c)t}{t(1-t)} \right] y' + \frac{ab}{t^2(1-t)} y = 0$  into the hypergeometric equation  $t(1-t)w'' + \{ (1+a-b) - [a + (1+a-c) + 1]t \} w' - a(1+a-c)w = 0$ . When  $a - b$  is not an integer, find two independent solutions for larger values of  $x$
9. Analyze the number of positive zeros of non-zero solutions of  $y'' + (k/x^2)y = 0$ , where  $k > 0$ .
10. Prove the following for a nontrivial solution  $y_p(x)$  of Bessel's equation:
- (a) If  $0 \leq p < 1$ , then every interval of length  $\pi$  contains at least one zero of  $y_p(x)$ .
- (b) If  $p = \frac{1}{2}$ , then the distance between successive zeros of  $y_p(x)$  is exactly  $\pi$ .
- (c) If  $p > \frac{1}{2}$ , then every interval of length  $\pi$  contains at most one zero of  $y_p(x)$ .
11. Let  $Q_n(x) = \frac{P_n(x)}{c}$ , where  $c$  is the leading coefficient of Legendre polynomial  $P_n(x)$ . Prove that among all polynomials of degree  $n$ ,  $Q_n(x)$  deviates least from zero on the interval  $-1 \leq x \leq 1$  in the sense of least squares.

(3 × 7 = 21 marks)

### Part C

Answer any three questions.

Each question carries 13 marks.

12. (a) Solve by power series method :  $y' - y = 0$
- (b) Determine whether  $x = 0$  is an ordinary point or regular singular point of the differential equation  $2x^2y'' + 7x(x+1)y' + 3y = 0$
13. (a) What is the  $n^{\text{th}}$  Legendre Polynomial?
- (b) State and prove the orthogonality conditions of sequence of Legendre polynomials on the interval  $[-1, 1]$
14. (a) Obtain  $J_p(x)$ , the Bessel function of the first kind of order  $p$ . What is the  $n^{\text{th}}$  Legendre Polynomial?
- (b) Show that  $(n + \frac{1}{2})! = \frac{(2n+1)!}{2^{2n+1}n!} \sqrt{\pi}$
15. (a) Find the general solution of the system:

$$\frac{dx}{dt} = 3x - 4y$$

$$\frac{dy}{dt} = x - y$$

- (b) Let  $W(t)$  be the Wronskian of the two solutions  $x = x_1(t); y = y_1(t)$  and  $x = x_2(t); y = y_2(t)$  of the system

$$\frac{dx}{dt} = a_1(t)x + b_1(t)y$$

$$\frac{dy}{dt} = a_2(t)x + b_2(t)y$$

Show that  $w(t)$  is either identically zero or nowhere zero on the specified interval.

16. Let  $f(x, y)$  be a continuous function that satisfies Lipschitz condition on the infinite strip  $a \leq x \leq b; -\infty < y < \infty$ . If  $(x_0, y_0)$  is any point of the strip, then show that  $y' = f(x, y); y(x_0) = y_0$  has a unique solution on the interval  $a \leq x \leq b$ .

(3 × 13 = 39 marks)