KANNUR UNIVERSITY

(Abstract)

M.Sc Mathematics Programme -Revised Scheme, Syllabus and Model Question Papers - Core/Elective Courses under Credit Based Semester System-Affiliated Colleges -Implemented with effect from 2014 Admission - Orders issued.

ACADEMIC BRANCH

U.O No. Acad/C4/7538/2014 Dated, Civil Station (PO), 24-07-2014

Read: 1. U.O.No.Acad/C1/11460/2013 dated 12/03/2014.
2. Minutes of the meeting of the Board of Studies in Mathematics (PG) held on 21-01-2014.
3. Minutes of the meeting of the Faculty of Science held on 25-03-2014.
4. Letter dated 18-06-2014 from the Chairman, Board of Studies in Mathematics (PG).

ORDER

1. The Revised Regulations for Credit Based Semester System for PG programmes in affiliated colleges have been implemented in this University with effect from 2014 admission vide paper read (1) above.

2. The Board of Studies in Mathematics (PG), vide paper read (2)above, has finalized the Scheme, Syllabus and Model Question Papers for M.Sc Mathematics under Credit Based Semester System with effect from 2014 admission.

3. As per the paper read (3) above, the meeting of Faculty of Science approved the Scheme, Syllabus and Model Question Papers for M.Sc Mathematics w.e.f.2014 admission.

4. The Chairman, Board of Studies in Mathematics (PG) vide paper (4) above, has forwarded the Scheme, Syllabus and Model Question Papers for M.Sc Mathematics for implementation with effect from 2014 admission.

5. The Vice Chancellor after considering the matter in detail and in exercise of the powers of Academic Council conferred under section 11 (1) of Kannur University Act 1996 and all other enabling provisions read together with has accorded sanction to implement Scheme, Syllabus and Model Question Papers (Core/Elective Courses) for M.Sc Mathematics Programme under Credit Based Semester System with effect from 2014 admission subject to report Academic Council.

6. The Implemented Scheme, Syllabus and Model Question Papers are appended.

7. Orders are, therefore, issued accordingly.

Sd/-

DEPUTY REGISTRAR (ACADEMIC)

For REGISTRAR

To
The Principals of Colleges offering M.Sc Mathematics Programme.

(PTO)

Copy to:

1. The Examination Branch (through PA to CE).
2. The Chairman BOS in Mathematics (P G).
3. PS to VC/PA to R/PA to CE
4. DR/AR 1 (Acad).
5. SF/DF/FC.

Forwarded /By Order

SECTION OFFICER

KANNUR UNIVERSITY KANNUR PIN-670567 Date:........... MANGATTUPARAMBA

For more details; log on www.kannur university .ac.in

SYLLABUS

M.Sc MATHEMATICS
KUCBSS Scheme

KANNUR UNIVERSITY

2014 ADMISSION

KANNUR UNIVERSITY
M.SC DEGREE PROGRAMME IN MATHEMATICS (KUCBSS)
SCHEME AND SYLLABUS (2014 ADMISSION)

COURSE STRUCTURE:

Course Code	Course Title	Lecture Hours/Week	Duration of Examination (Hours)	Credits	Marks
FIRST SEMESTER					
MAT1C01	Basic Abstract Algebra	5	3	4	75
MAT1C02	Linear Algebra	5	3	4	75
MAT1C03	Real Analysis	5	3	4	75
MAT1C04	Basic Topology	5	3	4	75
MAT1C05	Differential Equations	5	3	4	75
			Total	20	375
SECOND SEMESTER					
MAT2C06	Advanced Abstract Algebra	5	3	4	75
MAT2C 07	Measure and Integration	5	3	4	75
MAT2C08	Topology	5	3	4	75
MAT2C09	Complex Analysis	5	3	4	75
MAT2C10	Partial Differential Equations& integral equations	5	3	4	75
			Total	20	375
THIRD SEMESTER					
MAT3C11	Number Theory	5	3	4	75
MAT3C12	Functional Analysis	5	3	4	75
MAT3C13	Complex Function Theory	5	3	4	75
MAT3C14	Advanced Real Analysis	5	3	4	75
MAT3E01	Elective-1	5	3	4	75
			Total	20	375
FOURTH SEMESTER					
MAT4C15	Operator Theory	5	3	4	75
MAT4C16	Differential Geometry	5	3	4	75
MAT4E02	Elctive-2	5	3	4	75
MAT4E03	Elective-3	5	3	4	75
MAT4Pr01	Project Work	5	-	2	35
MAT4C17	Viva-Voce	-	-	2	40
			Total	20	375

Total Marks: 1500

Total Credits: 80

The students may choose one elective from each of the following.

Elective 1

1. MAT3E01 Graph Theory
2. MAT3E02 Probability Theory

Elective 2

3. MAT4E03 Calculus of Variations
4. MAT4E04 Commutative Algebra

Elective 3

5. MAT4E05 Fourier and Wavelet Analysis
6. MAT4E06 Operations Research

CONTINUOUS ASSESSMENT (CA)

This assessment shall be based on predetermined transparent system involving periodic written tests, assignments, seminars and attendance in respect of theory course and based on tests, lab skill, records/viva and attendance in respect of practical course.

The percentage of marks assigned to various components for internal evaluation is as follows.

THEORY

	Components	% of internal marks
i	Two test papers	40
ii	Assignments and Viva	20
iii	Seminars/Presentation of course study	20
iv	Attendance	20

To ensure transparency of the evaluation process, the internal assessment marks awarded to the students in each course in a semester shall be published on the notice board at least one week before the commencement of external examination. There shall not be any chance for improvement for internal marks.

The course teacher shall maintain the academic record of each student registered for the course, which shall be forwarded to the University, through the college Principle, after endorsed by the HoD,

TESTS

For each course there shall be at least two class tests during a semester. The probable dates of the tests shall be announced at the beginning of each semester. Marks should be displayed on the notice board. Valued answer scripts shall be made available to the students for perusal within 10 working days form the date of the tests.

ASSIGNMENTS

Each student shall be required to do 2 assignments for each course. Assignments after valuation must be returned to the students.

SEMINAR

Each student shall deliver one seminar as an internal component for every course and must be evaluated by the respective teacher in terms of structure,

QUESTION PAPER PATTERN

Each question paper all semesters will have Two Parts, Part A and Part B, Part A will have six short answer questions out of which four are to be answered. The questions are to be evenly distributed over the entire syllabus. Each question carries 3 marks. Part B will have there units, each unit consists of three questions from the respective three units of the syllabus. Four questions are to be answered form B without omitting any unit. Each question carries 12 marks.

Project Work:

At the end of the Fourth Semester every student is required to submit three copies of neatly typed dissertation based on the project work carries out under the guidance of teacher. The topic of the project work must be chosen from any area in Mathematics, which is a not already covered in the syllabus prescribed for MSc Programme of Kannur University. The project work has to be evaluated by two external examiners, followings the guidelines given below:

Components of Evaluation of Project Work:

	Components	Weightage
a	Content	1
b	Methodology	1
c	Presentation	2
d	Viva-Voce	1

Project:-

i) Arrangement of contents

The project should be arranged as follows-

1. Cover page and title page
2. Bonafide certificate/s
3. Declaration by the student
4. Acknowledgement
5. Table of contents
6. List of tables
7. List of figures
8. List of symbols, Abbreviations and Nomenclature
9. Chapter
10. Appendices
11. Reference:

ii) Page dimension and typing instruction

The dimension of the Project report should be in A4 size. The project report should be printed in bond paper and bound using flexible cover of the thick white art paper or spiral binding. The general text of the report should be typed with 1.5 line spacing. Paragraph should be arranged in justified alignment with margin 1.25" each on top. Left and right of the page with portrait orientation. The content of the report shall be around 40 pages.

iii) <u>A typical specimen of Bonafide Certificate</u>

KANNUR UNIVERSITY

<Font style Times New Roman-size 18>

BONAFIDE CERTIFICATE

<Font style Times New Roman-Size 16>

<Font Style Times New Roman-size 14>

Certificate that this project report "………………TITLE OF THE PROJECT…………" is the bonafide work of "………………………NAME OF THE CANDIDATE…………" who carried out the project work under my supervision.

<<signature of HoD>> <<signature of Supervisor/Co-supervisor>>

SIGNATURE	SIGNATURE
<<Name>>	<<Name>>
HEAD FO THE DEPARTMENT	SUPERVISOR
<<Academic Designation>>	<<Academic Designation>>
<<Department>>	<<Department>>
<<Seal with full address of the Dept.& college>>	<<Seal with full address>>

iv)Declaration by the student

DECLARATION

I,……………………….., hereby declare that the Project work entitled……………..(Title of the Project), ……………………………………………………has been prepared by me and submitted to Kannur University in partial fulfillment of requirement for the award of Bachelor of ……………..is a record of original work done by me under the supervision of Dr./Prof………………..of Department of ………………….college/(Name of Institute).

I also declare that this Project work has not been submitted by me fully or partly for the award of any Degree, Diploma, Title or recognition before any authority.

Place

Date Signature of the student

(Reg. No)

ATTENDANCE

The students admitted in the P.G programme shall be required to attend at least 75% percent of the total number of classes (theory/practical) held during each semester. The students having less than prescribed percentage of attendance shall not be allowed to appear for the University examination.

Condonation of shortage of attendance to a maximum of 10% of the working days in a semester subject to maximum of two times during the whole period of post graduate programme may be granted by the Vice-Chancellor of the University. Benefit of Cnndonation of attendance will be granted to the students on health grounds, for participating in University Union activities, meeting of the University bodies and participation in other extracurricular activities on production of genuine supporting documents with the recommendation of the Head of the Department concerned. A student who is not eligible for such Condonation shall repeat the course along with the subsequent batch.

Student who complete the course and secure the minimum required attendance for all the course of a semester and register for the University examinations at the end of the semester alone will be promoted to higher semester.

The students who have attendance within the limit prescribed, but could not register for the examination have to apply for the ***token registration***, within two weeks of the commencement of the next semester.

Attendance of each course will be evaluated (internally) as below:-

Attendance	**% of marks for attendance**
Above 90% attendance	100
85 to <90%	80
80 to <85%	60
76 to <80%	40
75%	20
<75	Nil

EVALUATION AND GRADING

The evaluation scheme for each course (including projects) shall contain two parts; (a) Continuous Assessment(CA) and (b) End Semester Evaluation (ESE). **20%** marks shall be given to CA and the remaining **80%** to ESE. The ratio of marks between **internal and external is 1:4.** Both internal and external evaluation shall be carried out using marks with corresponding grades and grade points in **7-point indirect relative grading system**.

Viva-Voce:

A viva-voce examination, covering the entire programme, must be conducted by two external at the end of the Fourth Semester.

MAT1C01: BASIC ABSTRACT ALGEBRA

Text Book: John. B. Fraleigh – A First Course in Abstract Algebra (7th Edition), Narosa (2003)

Unit I

Direct Products and finitely generated Abelian Groups, Group Action on a Set, Applications of Sylow Theorems.
(Chapter-2: Section 11; Chapter-3: Section 16; Chapter-7: Sections 36, 37)

Unit II

Field of Quotients of the Integral Domain, Isomorphism Theorems, Series of Groups, Free Abelian Groups, Field of Quotients of the Integral Domain
(Chapter-4: Section 21, Chapter-7: Section 34, 35, 38).

Unit III

Ring of Polynomials, Factorization of Polynomials over a Field, Homomorphisms and Factor Rings, Prime and Maximal Ideals
(Chapter-4: Section 22, 23; Chapter-5: Section 26, 27).

Reference:

1. N. Herstein: Topics in Algebra.Wiley India Pvt. Ltd, 2006
2. D. S. Malik, John. N. Merdson, M. K. Sen: Fundamentals of Abstract Algebra Mc Graw-hill Publishing Co., 1996
3. Clark, Allen: Elements of Abstract Algebra. Dover Publications, 1984
4. David M. Burton: A First course in Rings and Ideals.Addison-Wesley Educational Publishers Inc., 1970
5. Joseph. A. Gallian: Contemporary Abstract Algebra. Narosa, 1999
6. M. Artin: Algebra Addison Wesley; 2nd edition, 2010

MAT1C02: LINEAR ALGEBRA

Text Book: Kenneth Hoffman & Ray Kunze; Linear Algebra; Second Edition, Prentice-Hall of India Pvt. Ltd

Unit I

Linear Transformations: Liner Transformations, The Algebra of Linear Transformations, Isomorphism, Representation of Transformation by Matrices, Linear Functional, The Double Dual The Transpose of a Linear Transformation.

(Chapter-3; Sections 3.1, 3.2,3.3, 3.4, 3.5, 3.6, 3.7)

Unit II

Elementary Canonical Forms: Introductions, Characteristic Values, Annihilating Polynomials Invariant Subspace, Simultaneous Triangulations& Simultaneous Diagonalisation.

(Chapter-6: Section 6.1, 6.2,6.3, 6.4, 6.5, 6.6)

Unit III

Elementary Canonical Forms: Invariant Direct Sums, The Primary Decomposition Theorem.

The Rational and Jordan Forms: Cyclic Subspaces and Annihilators, Cyclic Decomposition and the Rational Forms, The Jordan Forms.

Inner Product Spaces: Inner Products, Inner Product Spaces.
(Chapter-6: Sections 6.7, 6.8; Chapter-7: Sections: 7.1, 7.2, 7.3 (Omit Proof of the theorems in this (7.3) section); Chapter-8: Sections 8.1, 8.2)

Reference:

1. Stephen H. Friedberg, Arnold J Insel and Lawrence E. Spence: Linear Algebra: 4th Edition 2002: Prentice Hall.
2. Serge A Land: Linear Algebra; Springer
3. Paul R Halmos Finite-Dimensional Vector Space; Springer 1974.
4. McLane & Garrell Birkhoff; Algebra; American Mathematical Society 1999.
5. Thomas W. Hungerford: Algebra; Springer 1980
6. Neal H.McCoy & Thomas R.Berger: Algebra-Groups, Rings & Other Topics: Allyn & Bacon.
7. S Kumaresan; Linear Algebra A Geometric Approach; Prentice-Hall of India 2003.

MAT1C03: REAL ANALYSIS

Text Book I: Walter Rudin: Principles of Mathematical Analysis; 3rd Edition McGraw-Hill International

Text Book 2: T.M Apostol: Mathematical Analysis 2nd Edition; Narosa Publications (1973)

Unit-I

Basic Topology: Finite, Countable and Uncountable Sets, Metric Spaces, Compact Sets Perfect Sets, Connected Sets, Continuity: Limits of Functions, Continuous Functions, Continuity and Compactness, Continuity and Connectedness, Discontinuities, Monotonic Functions, Infinite limits and Limits at Infinity.

(Text Book1; Chapter-2, All sections: Chapter-4, All sections)

Unit-II

Differentiation: The derivative of Real Function, Mean Value Theorems, The Continuity of Derivatives, L 'Hospital' s Rule, Derivatives of Higher Order Taylor's Theorem, Differentiation of Vector-Valued Functions. The Riemann-Stieltjes Integral: Definition and Existence of the Integral, Properties of the Integral.

(Text Book 1: Chapter-5; All sections; Chapter-6; sections 6.1 to 6.19)

Unit-III

The Riemann-Stieltjes Integral (Continued); Integration and Differentiation, Integration of Vector-Valued Functions,

(Text Book 1: Chapter-6; Sections 6.20 to 6.25;)

Functions of Bounded Variations and Rectifiable Curves.

(Text Book2; Chapter-6; Sections 6.1 to 6.12)

Reference:

1. R.G Bartle and D.R Sherbert; Introduction to Real Analysis; John Wiley Bros. 1982
2. L.M Graves; The Theory of functions of real variable; Tata McGraw-Hill Book Co.
3. M.H Porter and C.B Moraray; A first Course in Real Analysis; Springer Verlag UTM 1977.
4. S.C Sexena and S.M Shah: Introduction to Real Variable Theory, Intext Educational Publishers, San Francisco
5. S.R Ghopade and B.V Limaye; A Course in Calculus and Real Analysis, Springer.
6. N.L Carothers- Real Analysis Cambridge University Press.

MAT1C04: BASIC TOPOLOGY

Text: C. Wayne Patty, *Foundations of Topology*, Second Edition – Jones & Bartlett India Pvt. Ltd., New Delhi, 2012.

Unit – I

Topological Spaces: The Definition and Examples, Basis for a Topology, Closed Sets, Closures and Interiors of Sets, Metric spaces, Convergence, Continuous functions and Homeomorphisms.
[Chapter 1 Sections 1.2 to 1.7]

Unit – II

New spaces from old ones: Subspaces, The Product Topology on X x Y, The Product Topology, The Weak Topology and the Product Topology, The Uniform Metric, Quotient Spaces.
[Chapter 2 all sections]

Unit – III

Connectedness and Compactness: Connected spaces, Pathwise and local connectedness, Totally disconnected space, Compactness in metric spaces, Compact spaces,
[Chapter 3 all sections, Chapter 4 Sections 4.1 and 4.2]

References:

1. K. D. Joshi, *Introduction to General Topology*, New Age International (P) Ltd., Publishers.
2. Dugundji, *Topology*, Prentice Hall of India.
3. G. F. Simmons, *Introduction to Topology and Modern Analysis*, Mc Graw Hill.
4. S. Willard, *General Topology* , Addison Wesley Publishing Company.
5. J. R. Munkres, *Topology: A First Course*, Prentice Hall of India.
6. Murdeshwar M. G., *General Topology*, second edition, Wiley Eastern.
7. Kelley, *General Topology*, van Nostrand, Eastern Economy Edition.

MAT1C05: DIFFERENTIAL EQUATIONS

Text Book: G.F Simmons: Differential Equations with Historical Notes; Tata McGraw Hill.

Unit I

Power Series Solutions and Special Functions

Introduction. A Review of Power Series
Series Solutions of First Order Equations
Second Order Linear Equations. Ordinary Points
Regular Singular Points
Regular Singular Points (Continued)
Gauss's Hyper geometric Equation
The Point at Infinity
(Chapter-5; Sections 26 to 31)

Unit II

Legendre Functions, Bessel Functions and System of first order equations

Legendre Polynomials
Properties of Legendre Polynomials
Bessel Functions. The Gamma Function
Properties of Bessel functions
General Remarks on Systems
Linear Systems
Homogeneous Linear Systems with Constant Coefficients
Nonlinear Systems. Volterra's Prey-Predator Equations

(Chapter-8; Sections 44 to 47; Chapter-10; Sections 54 to 57)

Unit III

Non Linear Equations, The Existence and Uniqueness of Solutions

Autonomous Systems. The Phase Plane and Its Phenomena
Types of Critical Points. Stability
Critical Points and Stability for Linear Systems
Stability b y Liapunov's Direct Method
The Method of Successive Approximations 538
Picard's Theorem 543
Systems. The Second Order Linear Equation

(Chapter-11; Sections 58 to 61; Chapter-13; Sections 68 to 70)

Reference:

1. G.Birkoff and G.C Rota: Ordinary Differential Equations; Wiley and Sons; 3rd Edition (1978)
2. E.A Coddington; An Introduction to Ordinary Differential Equations; Prentice Hall of India, New Delhi (1974)
3. P.Hartmon; Ordinary Differential Equations; John Wiley and Sons
4. Chakraborti; Elements of Ordinary Differential Equations and Special Functions; Wiley Eastern Ltd New Delhi (1990)
5. L.S Poutrigardian: A Course in Ordinary Differential Equations; Hindustan Publishing Corporation Delhi (1967)
6. S.G Deo & V.Raghavendra; Ordinary Differential Equations and Stability Theory; Tata McGraw Hill New Delhi (1967)
7. V.I Arnold; Ordinary Differential Equations; MIT Press, Cambridge 1981.

MAT2C06: ADVANCED ABSTRACT ALGEBRA

Text Book: John. B. Fraleigh, A First Course in Abstract Algebra (7th Edition), Narosa (2003)

Unit I

Unique Factorization Domains, Euclidean Domains, Gaussian Integers and Multiplicative Norms, Introduction to Extension Fields (Chapter-9: Section - 45, 46, 47 and Chapter-6: Section - 29).

Unit II

Algebraic Extensions, Geometric Constructions, Finite Fields, Automorphisms of Fields. (Chapter-6: Section - 31, 32, 33 and Chapter-10 : Section- 48).

Unit III

The Isomorphism Extension Theorem, Splitting Fields, Separable Extensions. Galois Theory (Chapter-10: Section – 49, 50, 51, 53).

Reference:

1. I. N. Herstein: Topics in Algebra. Wiley India Pvt. Ltd, 2006
2. D. S. Malik, John. N. Merdson, M. K. Sen: Fundamentals of Abstract Algebra Mc Graw-hill Publishing Co., 1996
3. Clark, Allen: Elements of Abstract Algebra. Dover Publications, 1984
4. David M. Burton: A First course in Rings and Ideals. Addison-Wesley Educational Publishers Inc., 1970
5. Joseph. A. Gallian: Contemporary Abstract Algebra. Narosa, 1999
   M. Artin: Algebra Addison Wesley; 2nd edition, 2010

MAT2C07: MEASURE AND INTEGRATION

Text Book; G de Barra, Measure Theory and Integration. New age International Publishers, New Delhi (First Edition, 1981)

Unit I

Measure on the real line; Lebesgue Outer measure, Measurable sets, Ragularity, Measurable Functions, Borel and Lebesgue Measurability (Including Theorem 17),
Integration of functions of a Real Variable; Integration of Non-negative Functions.

(Chapter-2; Section 2.1-2.5, Chapter-3-Section 3.1)

Unit II

Integration of functions of a Real Variable; The general Integral, Riemann and Lebesgue Integrals
Abstract Measure Space; Measures and Outer measures, extension of measure, Uniqueness of the extension.

(Chapter-3, Section 3.2 and 3.4; Chapter-5; Section 5.1 –5.3)

Unit III

Abstract Measure Spaces; Measure Spaces, Integration with respect to a Measure
Inequalities and the L^P Spaces; The L^P Spaces, The inequalities of Holder and Minkowski, Completeness of $L^P(\mu)$

(Chapter-5, Section 5.5 –5.6; Chapter-6-section 6.1, 6.4 and 6.5)

Reference:

1. Walter Rudin; Real and Complex Analysis; 3rd Edition, Tata McGraw Hill
2. P.R Halmos; Measure Theory; D.Van Nostrand Co.
3. A.E Taylor; General Theory of Functions and Integrations; Blaisadel Publishing Company, New York
4. Inder k Rana; An Introduction to Measure and Integration; Narosa Publishing House, New Delhi. 1997.
5. Royden H.L Real Analysis Macmillan & Co
6. N.L Carothers-Real Analysis Cambridge Press.

MAT2C08 : TOPOLOGY

Text: C. Wayne Patty, Foundations of Topology, Second Edition – Jones & Bartlett India Pvt. Ltd., New Delhi, 2012.

Unit – I

The Separation and Countability Axioms: T0, T1 & T2 spaces, Regular and completely regular spaces, Normal and completely normal spaces, The countability axioms, Urysohn's Lemma and Tietze Extension Theorem, Embeddings.
[Chapter 5 all sections]

Unit – II

Compactness: Local Compactness and the Relation Between Various Forms of Compactness, The weak topology on a topological space, Equicontinuity.
Special Topics: Compactifications, The Alexander Subbase and the Tychonoff Theorems.
Metrizability and Paracompactness: Urysohn's Metrization Theorem.
[Chapter 4 Sections 4.3 to 4.5; Chapter 6 Sections 6.6 and 6.7; Chapter 7 Section 7.1]

Unit – III

The Fundamental Group and Covering Spaces: Homotopy of paths, The Fundamental Group, The Fundamental Group of the Circle, Covering Spaces.
[Chapter 8 Sections 8.1 to 8.4]

References:

1. K. D. Joshi, Introduction to General Topology, New Age International (P) Ltd., Publishers.
2. Dugundji, Topology, Prentice Hall of India.
3. G. F. Simmons, Introduction to Topology and Modern Analysis, Mc Graw Hill.
4. S. Willard, General Topology , Addison Wesley Publishing Company.
5. J. R. Munkres, Topology: A First Course, Prentice Hall of India.
6. Murdeshwar M. G., General Topology, second edition, Wiley Eastern.
7. Kelley, General Topology, van Nostrand, Eastern Economy Edition.

MAT2C09: COMPLEX ANALYSIS

Text Book: John B Conway- Functions of One Complex Variable, 2nd Edition, Springer International Student Edition

Unit I

Analytic Functions

Complex Integration

Power Series representation of Analytic Functions
Zeroes of an analytic function
The index of a closed curve
Cauchy's Theorem and Integral Formula
The homotopic version of Cauchy's Theorem and simple connectivity
Counting zeros the Open Mapping Theorem
Goursat's Theorem

Chapter III Section 2
IV Section 2 to 8

Unit II

Singularities

Classification of singularities
Residues
The Argument Principle

The Maximum Modulus Theorem

The Maximum Principle
Schwarz's Lemna

Chapter V Sections 1,2,3
VI Sections 1,2

Unit III

Compactness and Convergence in the Space of Analytic Functions

The Spaces of continuous functions $C(G, \Omega)$
Spaces of analytic functions
The Riemann Mapping Theorem
Weierstrass Factorization Theorem
Factorization of the sine function
The gamma function

Chapter VII Section 1 to 7 (except 3)

Reference:

1. Louis Pennise: Elements of Complex Variable Half, Richart & Winston 1976
2. Silverman.H: Complex Variable, Haughton Miffin Complex, Boston 1975.
3. Rudin.W: Real and Complex Analysis (3rd Edition) McGraw Hill International Edition 1967.
4. E.T Copson: An Introduction to the Theory of a Complex Variables, Oxford University Press 1974.
5. Lars V.Ahlfors-Complex Analysis (3rd Edition), Mc Graw-Hall international edition

MAT2C10: PARTIAL DIFFERENTIAL EQUATIONS AND INTEGRAL EQUATIONS

Text Book: 1. Amarnath M: Partial Differential Equations, Narosa, New Delhi(1997)
2. Hildebrand F.B: Methods of Applied Mathematics, (2nd Edition) Prentice- Hall of India, New Delhi(1972)

UNIT I

First Order P.D.E.

Curves and Surfaces, Genesis of first order Partial Differential Equations, Classification of integrals, Linear equations of first order, Pfaffian differential equations, Compatible systems, Charpit's method, Jacobi's method, Integral surfaces passing through a given curve, Quasi linear equations, Non-linear First order Partial Differential Equations
[Sections 1.1 – 1.11. from the Text 1]
Omit the Proof of Theorem 1.11.1

UNIT II

Second Order P.D.E.

Genesis of second order Partial Differential Equations.
Classification of second order Partial Differential Equations.
One dimensional Wave Equation.
Vibrations of an infinite String , Vibrations of semi-infinite String, Vibrations of a String of Finite Length, Riemann's Method, Vibrations of a String of Finite Length (Method of Separation of Variables).
Laplace's Equation.
Boundary Value Problems, Maximum and Minimum Principles, The Cauchy Problem, The Dirchlet Problem for the Upper Half Plane, The Neumann Problem for the Upper Half Plane, , Laplace's Equation - Green's Function, The Dirchlet Problem for a Half Plane , The Dirchlet Problem for a Circle .
Heat Conduction Problem.
Heat Conduction - Infinite Rod Case, Heat Conduction – Finite Rod Case,
[Sections 2.1 – 2.8. from the Text 1. Omit sections 2.4.5 to 2.4.13]

UNIT III

Integral Equations.

Introduction ,Relation Between differential and Integral Equation, The Green's Function, Frdholm Equation With Separable Kernels, Illustrative Examples, Hilbert Schmidt Theory, Iterative Methods for Solving Equations of the Second Kind, The Neumann Series , Fredholm Theory.
[Sections 3.1 – 3.3, 3.6 – 3.11 from the Text 2]

Reference:

1. E.A. Coddington : An Introduction to Ordinary Differential Equations
   Printice Hall of India ,New Delhi (1974)
2. F. John : Partial Differential Equations
   Narosa Pub. House New Delhi (1986)
3. Phoolan Prasad & Renuka Ravindran: Partial Differential Equations
   Wiley Eastern Ltd New Delhi (1985)
4. J.N Sharma and Kehar Sing, Partial Differential Equations for engineering scientists
   Narosa Pub. House New Delhi.

MAT3C11: NUMBER THEORY

Text Book:

1. Tom M Apostol: Introduction to Analytic Number Theory; Springer International Student Edtion
2. D.M Burton: Elementary Number Theory (6^{th} Edition) Mc Graw Hill
3. lan Stewart and David Tall: Algebraic Number Theory and Fermal's last theorem (Third Edition) A K Peters Natick Massachussets

Unit I

The Fundamental theorem of Arithmetic: Introduction-Divisibility-Greatest common divisor-prime numbers- The fundamental theorem of arithmetic-The series of reciprocals of primes-The Euclidean algorithm-The greatest common divisor of more than two numbers.
(Text 1, Sectons1.1-1.8)

Arithmetical Functions and Dirichlet multiplication: Introduction- The Mobius function $\mu(n)$ –The Euler totient function φ (n) –The relation connecting μ and φ -the product formula for φ (n) –The Dirichlet product of arithmetical functions- Dirichlet inverses and Mobius inversion formula- The Mangolt function $\Lambda(n)$ –Multiplicative functions- Multiplicative functions and Dirichlet multiplication- The inverse of a completely multiplicative function- Liouville's function $\lambda(n)$- The divisor function $\sigma_{\alpha}(n)$.

(Text 1, Section 2.1-2.13)

Congruences: Definition and basic properties of congruences- Residue classes and complete residue system- Liner Congruences-Reduced residue system and the Euler- Fermat theorem-Polynomial congruences modulo p and Langrange's theorem- Applications of Langrange's theorem- Simultaneous linear congruences and Chinese Remainder theorem- Applications of Chinese remainder theorem- Polynomial congruences with prime power moduli.

(Text 1, Section 5.1-5.9)

Unit II

Quadratic Residues and Quadratic Reciprocity Law: Quadratic residues- Legendre's symbol and its properties- Evaluation of $(-1|p)$ and $(2|p)$ Gauss lemma-The quadratic reciprocity law –Applications of the reciprocity law – The Jacobi symbol- Applications to Diophantine equations.

(Text 1, Sections 9.1 –9.8)

Primitive Roots: The exponent of number mod m and primitive roots- Primitive roots and reduced residu; system- The nonexistence of primitive roots mod 2a for $\alpha\geq 3$- The existence of primitive roots mod p for odd primes p- Primitive roots and quadratic residues – The existence of primitive roots and P^a- The existence of primitive roots mod 2 P^a –The nonexistence of Primitive roots in the remaining cases- The number of primitive roots mod m.

(Text 1, Sections 10.1-10.9)

Introduction to Cryptography; From Caesar Cipher to Public Key Cryptography-The Knapsack Crypto system- An application of primitive roots to Cryptography.

(Text 2, Sections 10.1-10.3)

Unit III

Algebraic Backgrounds: Symmetric polynomials- modules- free abelian groups
(Text 3, Section 1.4-1.6)

Algebraic Numbers: Algebraic numbers- Conjugates and Discriminants- Algebraic integers- Integral bases- Norms and Traces- Rings of integers.
(Text 3, Section 2.1-2.6)

Quadratic and Cyclotomic fields: Quadratic fields-Cyclotomic fields.

(Text 3, Sections 3.1-3.2)

Reference:

1. G.H Hardy and E.M Wright: An introduction to the theory of numbers, Oxford University Press.
2. I Niven, H.S Zuckerman, H.L Montgomery; An Introduction to the theory of numbers, Wiley India
3. Emil Grosswald: Introduction to number theory.
4. P.Samuel; Theory of Algebraic Numbers, Herman Paris Haughton Mifflin
5. S.Lang Algebraic Number Theory Addison Wesley Pub. Co Reading.

MAT3C12: FUNCTIONAL ANALYSIS

Text Book; Balmohan V Limaye; Functional Analysis (2nd Edition); New Age International Publishers.

Unit I

Fundamentals of Normed Spaces; Normed Spaces, Banach spaces, Continuity of Linear Maps, Hahn-Banach Theorems.

(Chapter-2, Sections 5,6,7,8)

Unit II

Bounded Linear Maps on Banach Spaces; Uniform Boundedness Principle, Closed Graph and Open Mapping Theorems, Bounded Inverse Theorem

(Chapter-3, Section 9, 10, 11, Omit Quadrature Formula and Matrix Transformation and Summability Methods of Section 9)

Unit III

Geometry of Hilbert Spaces; Inner Product Spaces, Orthonormal Sets. Approximation and Optimization, Projection and Riesz Representation Theorems.

(Chapter-6, Section 21,22, 23, 24 (Omit 23.2, 23.6, 24.7, 24.8))

Reference:

1. E.Kreyszig; Introductory Functional Analysis with Applications, John Wiley
2. Walter Rudin; Functional Analysis, TMH Edtions 1978
3. M.T Nair; Functional Analysis A First Course; Prentice Hall of India.
4. Chaudhary and Sudarsan Nanda; Functional Analysis with Applications, Wiley Eastern Ltd.
5. Walter Rudin; Introduction to Real and Complex Analysis, McGraw Hill International Edition
6. J.B Conway; Functional Analysis, Narosa Publishing Company
7. Bachman and Narici; Functional Analysis

MAT3C13: COMPLEX FUNCTION THEORY

Text Book 1: Lars V.Ahlfors-Complex Analysis (3rd Edition), Mc Graw-Hall international edition

Text Book 2: John B Conway- Functions of One Complex Variable, 2nd Edition, Springer International Student Edition

Unit I

Elliptic Functions
- Simple periodic functions
- Doubly periodic functions

The Weierstrass Theory
- Chapter 7, Sections 1,2,3 of Text 1

The Riemann Zeta function
- Chapter 7, Sections 8 of Text 2

Unit II

Runge's Theorem Runge's
- Theorem Simple
- Connectedness
- Mittag Lefler's Theorem

Analytic Continuation and Riemann Surfaces
- Schwarz Reflection Principle
- Analytic Continuation along a path
- Mondromy Theorem
- (Chapter VIII, Section 1,2,3, of text 2
  IX Section 1,2,3 of text 2)

Unit III

Harmonic Functions
- Basic Properties of harmonic functions
- Harmonic functions on a disk
- Sub harmonic and super harmonic functions

Entire Functions
- Jensen's formula
- The genus and order of entire function
- Hadamard Factorization Theorem
- (Chapter X, Sections 1,2,3 of Text 2)
- (Chapter XI, Sections 1,2,3 of Text 2)

Reference:

1. Louis Pennise: Elements of Complex Variable Half, Richart & Winston 1976
2. Silverman.H: Complex Variable, Haughton Miffin Complex, Boston 1975.
3. Rudin.W: Real and Complex Analysis (3rd Edition) McGraw Hill International Edition 1967.
4. E.T Copson: An Introduction to the Theory of a Complex Variables, Oxford University Press 1974.

MAT3C14: ADVANCED REAL ANALYSIS

Text Book: Walter Rudin: Principles of Mathematical Analysis; (3rd Edition) Mc. Graw Hill, 1986.

Unit I

Sequence and series of Functions: Discussion of Main Problem, Uniform Convergence, Uniform Convergence Continuity, Uniform Convergence and Integration, Uniform Convergence and Differentiation, Equicontinous Family of Functions, The Stone-Weierstrass Theorem,

(Chapter-7; Sections 7.1 to 7.33 and Theorem 7.33)

Unit II

Some Special Functions; Power Series, The Exponential and Logarithmic Functions, The Trigonometric Functions, The Algebraic Completeness of the Complex Field, Fourier Series. The Gamma Function

(Chapter-8: Sections 8.1 to 8.22)

Unit III

Functions of Several Variables: Liner Transformations, Differentiation The Contraction Principle, The Inverse Function Theorem, The Implicit Function Theorem,

(Chater-9; Sections 9.1 to 9.29)

Reference:

1. R.G Bartle and D.R Sherbert; Introduction to Real Analysis; John Wiley Bros. 1982
2. L.M Graves; The Theory of Functions of a Real Variable; Tata McGraw-Hill Book Co 1978
3. M.H Protter and C.B Moray; A First course in Real Analysis; Springer Verlag UTM 1977
4. T.M Apostol; Mathematical Analysis; 2nd Edition; Narosa Publications 1973.

MAT4C15: OPERATOR THEORY

Text Book: Balmohan V Limaye; Functional Analysis(2nd Edition); New Age International Publishers

Unit I

Spectrum of a Bounded Operator-Spaces of Bounded Linear Functionals; Duals and Transposes Weak and Weak* Convergence

(Chapter-3 Section-12; Chapter-4 Sections 13; 13.1 to 13.6 and Sections 15; 15.1 to 15.4)

Unit II

Spaces of Bounded Linear Functionals; Reflexivity, Compact Operators on Normed Spaces: Compact Linear Maps, Spectrum of a Compact Operator.

(Chapter-4, Section 16; Chapter-5, Sections 17,18)

Unit III

Bounded Operators on Hilbert Spaces; Bounded Operators and Adjoints, Normal, Unitary and Self Adjoint Operators, Spectrum and Numerical Range, Compact Self Adjoint Operators.

(Chapter-7; Section 25, 26(omit 26.6) and 27and 28; 28.1 to 28.4 and 28.5 Statement only)

Reference:

1. E.Kreyszig; Introductory Functional Analysis with Applications, John Wiley
2. Walter Rudin; Functional Analysis, TMH Edtion 1978.
3. M.T Nair: Functional Analysis A First Course: Prentice Hall of India
4. Chaudhary and Sudarsan Nanda: Functional Analysis with Applications, Wiley Eastern Ltd.
5. Walter Rudin: Introduction to Real and Complex Analysis, McGraw Hill International Edition
6. J.B Conway: Functional Analysis, Narosa Publishing Company
7. Bachman and Narici; Functional Analysis.

MAT4C16: DIFFERENTIAL GEOMETRY

Text Book: John A Thorpe: Elementary Topics in Differential Geometry, Springer Verlag NY Heidelberg, Berlin

Unit I

Graphs and Levels Sets, Vector Fields, The Tangent Space, Surfaces, Vector fields on Surfaces, Orientation

(Chapter 1,2,3,4,5)

Unit II

The Gauss map, Geodesics, Parallel Transport, The Weingarten Map, Curvature of Plane Curves.

(Chapter 6,7,8,9,10)

Unit III

Are Length and Line Integrals, Curvature of Surfaces, Parameterized Surfaces, and Local Equivalence of Surfaces and Parameterized Surfaces.

(Chapter 11,12,14,15)

Reference:

1. W I Burko: Applied Differential Geometry, Cambridge University Press (1985)
2. M.De Carmo: Differential Geometry of Curves, Surfaces (Prentice Hall Inc. Englewood cliffs N.J (1976)
3. V. Grilleman and Pollack: Differential Topology, Prentice Hall, Inc Englewood cliffs N.J (1974)
4. Singer and J.A Thorp: Lecture notes on elementary Topology and Geometry CUTM Springer Verlag, New York (1967)
5. R. Millmen and Parker: Elements of Differential Geometry (Prenice Hall Inc. Englewood cliffs N.J (1977)
6. M Spivak: A Comprehensive Introduction to Differential Geometry, Vol 1 to 5, Perish Boston (1970-75)

MAT3E01:
Elective

Graph Theory (Elective)

Text 1 J.A Bondy and U.S Murty, Graph Theory with Applications, The MacMillan Press Ltd, 1976

Text 2 John Clark and Derek Allan Holtan, A First Look at Graph Theory, Allied Publishers, Ltd

Unit I

Independent Sets and Cliques; Independent Sets, Ramsey's Theorem, Turan's Theorem, Shur's Theorem

Vertex Colorings: Chromatic Number, Book's Theorem Hajo's Conjecture, Chromatic Polynomials, Girth and Chromatic Number.

(Chapter 7; Except Section 7.5, Chapter 8 Except Section 8.6, Text 1)

Unit II

Edge Colourings: Edge Chromatic Number, Vizing's Theorem, The Timetabling Problem

Planar Graphs; Plane and Planar Graphs, Dual Graphs, Euler's Formula Bridges, Kuratowski's Theorem. The Five Colour Theorem Non Hamiltonian Planar Graphs.

(Chapter 6, All sections; Chapter 9; Except section 9.8 of Text 1)

Unit III

Matchings: Matchings, Matchings and Coverings in bipartite Graphs, Perfect Matchings,The Personnel Assignment Problem, The Optimal Assignment Problem.
(Chapter 5, Sections 5.1, 5.2, 5.3, 5.4, 5.5 of text 1)

Networks; Flows and Cuts, Separating sets

(Chapter 8; Sections 8.1 & 8.3 of text 2

Reference:

1. F. Harraray, Graph Theory, Narosa Publishing House.
2. Narasingh Deo, Graph Theory with applications to Engineering and Computer Science, PHI.
3. O.Ore, Graph and Their uses, Random House Inc, NY (1963)
4. K.D Joshi, Foundations of Discrete Mathematics, Wiley Eastern Ltd.

MAT3E02:
Elective

PROBABILITY THEORY

Text Book: A.K. Basu: Measure and Probability, Prentice Hall of India 2003.

Unit I

Sets and Sequences of Sets, Sequence of Sets, Fields and σ- Fields, Monotone Class, Borel Sets in Real line, Fundamental Properties of Measure Different Example of Measure, Random Variables and Measurable Transformations, Discrete Sample Space, Combinatorial Aspects of Set Functions.

(Chapter-1; All sections; Chapter-2; Sections 1,2,3,4 and 5)

Unit II

Induced Measure and Distribution Function, Probability Generating Function and Discrete Convolution, Continuity Theorem for Additive Set Functions and Applications, Distribution Function (Univariate), Multivariate Distribution Function, Stieltjes Integral, Characteristic Functions, Moments and Applications, Inversion Theorem, Continuity Theorem and their Applications. Test for Characteristic Functions and Polya's Theorem, Necessary and Sufficient Condition for a Characteristic Function.

(Chapter-2, Sections 6,7 and 8; Chapter-3; Sections 1,2,3,4,5,6,7 and 8)

Unit III

Weak Convergence and Helly's Theorems Algebra of Measurable Functions, Almost Every Where Convergence, Convergence in Measure Definition and Properties of Integration and Expectation, Moments and Inequalities, Kolomogorov's Definition of Expectation.

(Chapter-3; Section 9, Chapter-4, Section 1,2 and 3; Chapter-5; Sections 1,2 and 3(Excluding Sub Section 5.3.2))

Reference:

1. B.R Bhat; Modern Probability Theory, 2nd Edition, Wiley Eastern, New Delhi.
2. A.N Kolmogrov; Foundations of Probability, Chelsea, NY (1950)
3. Loeve.M Probability Theory, Van-Nostrand, Princeton (1963)
4. Chows. Y.S and Tiecher H; Probability Theory, Springer Verlag (1988).

MAT4E03:
Elective

CALCULUS OF VARIATIONS

Text Book:I M. Gelfand and S.V Fomin; Calculus of Variations, Prentice Hall Inc, N.Y (1963)

Unit I

Elements of the Theory, Further Generalizations

(Chapter-1, all Sections ; Chapter-2 all Section)

Unit II

General Variations of a Functional, The Canonical Form of the Euler Equations and related topics

(Chapter-3 All sections; Chapter-4 All sections)

Unit III

The Second Variation, Sufficient condition for a Weak Extremum

(Chapter-5 All sections)

Reference:

1. Bliss G.A Calculus of Variations, Open Court Publishing Co. Chicago (1925)
2. Bolza O Lecture on Calculus of Variations, G.E Stinchar & Co. NY (1931)
3. Courant R and Hilbert D; Methods of Mathematical Physics, Vol. 1 Wiley Eastern Reprint (1975)
4. Elsgoltz I; Differential Equations and Calculus of Variations, Mr Publishers Moscow (1973)
5. Morse M. The Calculus of Variations, American Mathematical Society (1934)

MAT4E04:
Elective

COMMUTATIVE ALGEBRA

Text Book: Atiyah M.F and Macdonald I.G; Introduction to communicative Algebra, Addison Wiley (1969)

Unit I

Rings and Ideals, Modules; Rings and Ring Homomorphism, Ideals, Quotient Rings, Zero Divisors, Nilpotent Elements, Unit, Prime Ideals and Maximal Ideals, Nilradical and Jocobson Redical, Operations on Ideals, Extension and Contraction, Modules and Module Homomorphism, Submodules and Quotient Modules, Operations on Submodules, Direct Sum and Product, Finitely Generated Modules, Exact Sequences.

(Chapter-1; All Sections; Chapter-2; Section 2.1 to 2.11)

Unit II

Rings and Modules of Fractions, Primary Decomposition: Local Properties, Extended and Contracted Ideals. Primary Decomposition.

(Chapter-3; All sections; Chapter-4; All Section)

Unit III

Integral dependence, Chain conditions, Noetherian Rings; Integral Dependence, The Going-Up Theorem, Integrally Closed Integral Domains. The Going-Down Theorem, Chain Conditions, Noetherian Rings.

(Chapter-5; All section, except 5.18, 5.19, 5.20, 5.21, 5.22, 5.23, 5.24; Chpater-6; All sections; Chapter-7; All sections, except 7.8, 7.9 and 7.10)

Reference:

1. N.Bourbaki: Commutative Algebra, Paris Herman (1961)
2. D.Burton; A first course introduction to Rings and Ideals, Wesley (1970)
3. N.S Gopalakrishnan; Commutative Algebra, Oxonian Press (1984)
4. T.W Hunger ford; Algebra, Springer Verlag (1974)
5. D.G North Cott; Ideal Theory, Cambridge University Press (1953)
6. O.Zarisiki and P. Samuel; Commutative Algebra, Vol I and II, Van Nostrand, Princeton (1960).

MAT4E05:
Elective

FOURIER AND WAVELET ANALYSIS

Text Book: M.W. Frazier, An Introduction to Wavelets through Linear Algebra; Springer (1999)

Unit I

Construction of Wavelets on Z_n, The First Stage.
Construction of Wavelets on Z_n,The Iteration Step.
The Haar System, the Shannon wavelets and the Daubechies's D6 wavelets on Z_n.
(Chapter-3, Sections 3.1 , 3.2 and Examples 3.32, 3.33 and 3.35 of Section 3.3.)

Unit II

l^2 (Z), Complete Orthonormal sets in Hilbert Spaces l^2 (Z) and Fourier Series,
The Fourier transforms and convolution on l^2 (Z).
First Stage Wavelets on Z, The Iteration Step for Wavelets on Z.
(Chapter-4, Sections 4.1 to 4.6)

Unit III

L^2 (R) and Approximate Identities.
The Fourier Transform on R.
(Chapter-5, Section 5.1 to 5.2)

References:

1.G.Bachman, L.Narici, E. Beckenstein : Fourier and Wavelet Analysis, Springer (2000)
2. I.Daubechies : Ten Lectures on Wavelets, SIAM (1992)
3. C.Heil : A Basis Theory Primer, Birkhauser (2011)
4. D.F Walnut : An Introduction to Wavelet Analysis, Birkhauser (2002)

MAT4E06:
Elective

OPERATIONS RESEARCH

Text Book; Kanti Swarup, P.K Gupta, Man Mohan; Operations Research; Sultan Chand & Sons. New Delhi (2007)

Unit I

Markov Analysis, Decision Analysis, Simulation

(Chapter-15; All Sections; Chapter-16; All Sections; Chapter-22; Section 22.1 to 22.9)

Unit II

Reliability and System failure rates, Inventory Control

(Chapter-18; Section 18.6, Chpater-19; All Sections, expect 19.8 and 19.9)

Unit III

Information Theory (Chpater-30; Section 30.1 to 30.10)

Reference:

1. K.V Mittal; Optimization methods on Operations Research and System Analysis, New Age International (P) Ltd. New Delhi
2. J.K Sharma; Operations Research-Theory and Applications, Macmillan, New Delhi
3. R.K Gupta; Operations Research, Krishna Prakashan Mandir II, Shivaji Road, Meerat-2,
4. L.R Potti; Operations Research, Yamuna Publications, Sreekanteswaram, Thiruvananthapuram
5. Premkumar Gupta and D.S Hira; Operations Research, S.Chand & Company Ltd. Ram Nagar New Delhi 1995.
6. B.S Goel and S.K Mittal; Operations Research, Pragti Prakashan Meerat-2

First Semester M.Sc Degree Examination
Mathematics
Model Question Paper 2014 Admissions
MAT1C01: Basic Abstract Algebra

Time: Three Hours Maximum : 60 Marks

Part A

Answer four questions from this part.

Each question carries 3 marks.

1. Are the groups $\mathbb{Z}_2 \times \mathbb{Z}_{12}$ and $\mathbb{Z}_4 \times \mathbb{Z}_6$ isomorphic ? Why or why not ?.
2. Let X be a G-set. Then prove that G_x is a subgroup for each $x \in X$
3. Prove that no group of order 20 is simple.
4. Is $\{(2,1),(4,1)\}$ a basis for $\mathbb{Z} \times \mathbb{Z}$? Prove your assertion.
5. Factorize $f(x) = x^4 + 3x^3 + 2x + 4$ in $\mathbb{Z}_5[x]$.
6. Find all $c \in \mathbb{Z}_3$ such that $\mathbb{Z}_3[x] / < x^2 + c >$ is a field.

Part B

Answer **4** questions from this part without omitting any Unit. Each question carries **12** marks.

UNIT I

7. (*a*) Prove that direct product of abelian groups is abelian.

   (*b*) Prove that $\mathbb{Z}_m \times \mathbb{Z}_n$ is isomorphic to $\mathbb{Z}_{mn}$ if and only if $\gcd(m,n) = 1$.

8. (*a*) Let X be a G-set and let $x \in X$. Then prove the following:

   (i) $|Gx| = (G : G_x)$

   (ii) If $|G|$ is finite then $|Gx|$ is a divisor of $|G|$

   (*b*) Prove Cauchy's theorem for groups.

9. (a) Prove that every group of prime power order is solvable.

(b) Prove that for a prime number p, every group of order p^2 is abelian.

UNIT I I

10. (a) Prove that any two fields of quotients of an integral domain D are isomorphic.

(b) Find the elements of $\mathbb{C}$ that makes up the field of quotient of

$D = \{n + mi : n, m \in \mathbb{Z}\}$ in $\mathbb{C}$

11. (a) State and prove second isomorphism theorem.

(b) Prove that two subnormal series of a group G have isomorphic refinements.

12. (a) If G is a nonzero free abelian group with a finite basis of r elements, then prove that G is isomorphic to $\mathbb{Z} \times \mathbb{Z} \times \cdots \times \mathbb{Z}$ for r factors.

(b) Prove that every finitely generated abelian group is isomorphic to a group of the form $\mathbb{Z}_{m_1} \times \mathbb{Z}_{m_2} \times \cdots \times \mathbb{Z}_{m_r} \times \mathbb{Z} \times \mathbb{Z} \times \cdots \times \mathbb{Z}$ where m_i divides m_{i+1} for $i = 1, 2, \cdots, r-1$.

UNIT I I I

13. (*a*) Prove Evaluation homomorphism theorem.

(*b*) Prove that $R[x]$ is a ring if R is a ring.

14. (*a*) Prove Eisenstein criteria.

(b) Prove that the cyclotomic polynomial is irreducible over $\mathbb{Q}$.

15. (*a*) Prove that R/M is a field if and only if M is a maximal ideal of R.

(*b*) Prove that and ideal $< p(x) > \neq \{0\}$ of $F[x]$ is maximal if and only if p(x) is irreducible over F.

First Semester M.Sc Degree Examination
Mathematics
Model Question Paper 2014 Admissions
MAT1C02: Linear Algebra

Time: 3hrs. Max.Mark:60

Part A
Answer four questions form this part
Each questions carries 3 marks

1. Does there exists a linear transformation T: $R^3 \rightarrow R^3$ such that T(1,0,1)=(1,1), T(-1,-1,0)= (1,2) and T(0,1,1)=(2,1) ? Why?

2. Let T: $R^3 \rightarrow R^3$ be defined by T(x,y)=(x+y, 2x). Find the matrix of T with respect to the usual basis of R^2.

3. Let V=R^3 and S= {(1,0,0),(1,1,0)}. Find the annihilator S^o?

4. Prove that if T is a linear operator on F^n such that T^2=T, where F is either C or R, then T is diagonalizable.

5. Prove that the T-Cyclic subspace z($\propto$:T) is one dimensional if and only if $\propto$ is a characteristic vector for T.

6. Let V be a inner product space and let x ε V prove that if <x,y>=0 for all yεV then x=0.

Part B
Answer any four questions from this part without omitting any unit.
Each question carries 12 marks.
Unit – I

7. a) Let V and W be vector spaces over the field F and let T be a linear transformation from V into W. Prove that if V is finite dimensional, then rank(T) + nullity T = dim V
   b) Find a basis for the space L(R^2, R^3) over R.

8. a) Let B and B' be two ordered bases for an n dimensional vector space V over the field F and T be a linear operator on V. Then prove that there exist an invertible nxn matrix P over F such That $[T]_{B'} = D^{-1}[T]_B P$

   b) Find range an null space of the linear operator T(a,b,c)=(a+b,2c,0) on R^3.

9. a) Let V be a finite dimensional vector space over a field F. Let {e_1,e_2,……., e_n}be a basis of V. Describe the dual basis and show that it is a basis for the dual space V*.
   b) Let V=R^3 over R. Give a basis for V and give the dual basis.

Unit- II

10. a) Prove the Cayley-Hamilton theorem.

b) Let A and B be nxn matrices over a field F. Prove that if I-AB is invertible then I-BA is also invertible. Deduce that AB and BA have the same characteristic values.

11. a) Let T be a linear operator on an n-dimensional vector space V. Then prove that the characteristic and minimal polynomials for T have same roots, except for multiplicities.

b) Fine the minimal polynomial for A= $\begin{pmatrix} 1 & 1 & 0 & 0 \\ -1 & -1 & 0 & 0 \\ -2 & -2 & 2 & 1 \\ 1 & 1 & -1 & 0 \end{pmatrix}$

12. a) State and prove a necessary and sufficient condition for a linear operator on a finite dimensional vector space to be triangularly.

b) Find an invertible matrix P such that P^{-1} AP is a diagonal matrix,

where A= $\begin{pmatrix} -1 & 1 \\ 1 & 1 \end{pmatrix}$

Unit III

13. a) State and prove primary decomposition theorem.

b) if $T(x_1,x_2)= (2x_1, x_2, -x_1)$ find a diagonalizable operator D and a nilpotent operator N on R^2 such that T=D+N

14. a) Define a cyclic vector for a linear operator T of a vector space. If a linear operator T of a finite dimensional vector space has a cyclic vector show that dim V is the same as the degree of the minimal polynomial of T.

b) Let T has a diagonalizable operator of n-dimensional vector space. If T has a cyclic vector show that T ahs r distinct characteristic values.

15. a) Define an inner product space. Prove that an orthogonal set of non-zero vector in an inner product space is linearly independent.

b) If W is a finite – Dimensional subspace of an inner product space V then show that $V=W\oplus W^{\perp}$ where is the orthogonal complement of $W^{\perp}$ in V.

First Semester M.Sc Degree Examination
Mathematics
Model Question Paper 2014 Admissions
MAT1C03: Real Analysis

Time: 3 hrs Max.Marks: 60

Part A

Answer four questions form this part
Each questions carries 3 marks

1. Verify whether $d(x,y) = \frac{|x-y|}{1+|x-y|}$ x,y in R, is a metric on R

2. Let f and g be continuous mappings of a metric space X into a metric space Y and E a dense subset of X. If f (p) = g(p) for all p in E, prove that f (p) = g (p) for all p in X.

3. Show that L' Hospital's rule does not hold for vector-valued functions.

4. If $f(x) = |x|^3$ compute $f^1(x)$, $f^2(x)$, for all real x. Show that $f^3(0)$ does not exists.

5. Examine whether the function given by $f(x) = x^2 \sin(\frac{1}{x})$, for $x \neq 0$), $f(0) = 0$ is of bounded variation on [0, 1]

6. $f \geq 0$ is continuous on [a, b] and $\int_a^b f = 0$ show that f = 0

Part B

Answer any four questions from this part without omitting any unit.
Each question carries 12 marks.

Unit – I

7. Show that for a subset E of R^k, following statements are equivalent
   i) E is closed and bounded
   ii) E is compact
   iii) Every infinite subset of E has a limit point in E.

8. a) Let P be a nonempty perfect set in R^k. Show that P is uncountable.
   b) Show that a subset E of the real lime R is connected if and only if x, y in E and x<z<y impies that z in E.

9. a) Show that a continuous function f from a compact metric space X into a metric space Y is uniformly continuous on X.

b) Let f be monotonic on (a, b). Show that the set of point of (a, b) at which f is discontinuous is at most countable.

Unit – II

10. a) State and prove the Taylor's Theorem

b) Let f be a continuous mapping of [a, b] into R^k which is differentiable in (a, b) show that there exists x in (a, b) such that $f|(b)-f(a)| \leq (b-a)|f^1(x)|$

1. a) State and prove the generalized mean value theorem.

b) If f is monotonic on [a, b] and α is continuous on [a, b] show that f is in R(α)

12. a) If f, g in R(α) show that f + g in R(α) and $\int (f + g)\, d\alpha = \int f d\alpha + \int g d\alpha$

b) Let f be of bounded variation on [a, b], α is monotonically increasing and α^1 in R[a, b]. Show that $\int f d\alpha = \int_a^b f(x)\alpha'(x)dx$

Unit – III

13. a) Let f:[a, b] → R^k and →f in R(α) for some monotonically increasing function α on [a, b]. Show that |f| in R(α) and $\left|\int_a^b f d\alpha\right| \leq \int_a^b |f|\, d\alpha$

b) Let f be continuous on [a,b] show that f is of bounded variation on [a,b] if and only if f can be expressed that difference of tow increasing continuous functions.

14. a) Let F and G be differentiable functions on [a, b] such that $F^1 = f$ in R and $G^1 = g$ in R. Show that

$$\int_a^b F(x)g(x)dx = F(b) \quad F(a)G(a) \quad \int_a^b f(x)G(x)\,dx$$

b) If f is of bounded variation on [a, b], show that f is bounded on [a, b]

c) Let f be a rectifiable path on [a, b] of are length $\Lambda_f(a,b)$ and c in (a, b). Show that $\Lambda_f(a,b) = \Lambda_f(a,c) + \Lambda_f(c,b)$

15. a) Let f be a monotonic function on [a, b]. Show that f is of bounded variation on [a. b]

b) Let f: [a, b] → R^n be a path with components $f = (f_1, f_2, \ldots\ldots\ldots\ldots f_n)$ Show that f is rectifiable if and only if each component f_k is of bounded variation on [a, b].

First Semester M.Sc Degree Examination
Mathematics
Model Question Paper 2014 Admissions
MAT1C04: BASIC T O P O L O G Y

Time: Three Hours | Maximum : 60 Marks

Part A

Answer **four** *questions from this part.*
Each question carries **3** *marks.*

1 Give six topologies on the set $\{1, 2, 3\}$.

2 Prove that in a metric space every convergent sequence is a Cauchy sequence. Is the converse true? Justify your answer with an example.

3 Let $X = \{1, 2, 3\}$, $\tau = \{\emptyset, \{1\}, \{1, 2\}, X\}$, $Y = \{4, 5\}$, and $U = \{\emptyset, \{4\}, Y\}$. Give a basis for the product topology on $X \times Y$.

4 For each natural number number *n*, let $X_n = R$ and let τ_n be the discrete topology on X_n. Let τ be the product topology on $X = \prod_{n \in N} X_n$. Is τ the discrete topology on X? Either prove that it is or show by example that it is not.

5 Let (X, τ) be a topological space and define a relation $\sim$ on X by $x \sim y$ provided there is a pathwise connected subset A of X such that $x, y \in A$. Prove that $\sim$ is an equivalence relation on X.

6 Let A be a subset of a topological space (X, τ). Prove that A is compact if and only if every cover of A by members of τ_A has a finite subcover.

Part B

Answer **four** *questions from this part without omitting any Unit.*
Each question carries **12** *marks.*

UNIT I

7 (*a*) Prove that the topology generated by the square metric on R^2 is the usual topology.

(*b*) Describe cocountable topology.

(*c*) State and prove the necessary and sufficient condition for a subset of the power set $P(X)$ to be a basis for a topology on X.

[4 + 4 + 4 = 12 marks]

8(*a*) Let $X = \{1, 2, 3, 4, 5\}$. Find the topology with $S = \{\{1\}, \{1, 2, 3\}, \{2, 3, 4\}, \{3, 5\}\}$ as a subbasis.

(*b*) Prove that every metric space is first countable.

(*c*) Let τ be the usual topology on R. Prove that $B = \{(a, b) : a < b \text{ and } a \text{ and } b$ are rational} is a countable basis for τ .

[4 + 4 + 4 = 12 marks]

9(*a*) Prove that a subset A of a topological space (X, τ) is a perfect set if and only if it is closed and has no isolated points.

(*b*) Let (X, d) be a metric space such that every Cauchy sequence in X has a convergent subsequence. Then prove that (X, d) is complete.

(*c*) Prove that metrizability is a topological property.

[4 + 4 + 4 = 12 marks]

UNIT I I

10(*a*) Prove that every subspace of a separable metric space is separable.

(*b*) Define product topology of two topological spaces. Give an example.

(*c*) Let (X, τ) , (Y_1, U_1) and (Y_2, U_2) be topological spaces, let $f_1 : X \to Y_1$ and $f_2 : X \to Y_2$ be functions, and define $f : X \to Y_1 \times Y_2$ by $f(x) = (f_1(x), f_2(x))$. The show that f is continuous if and only if f_1 and f_2 are continuous.

[4 + 4 + 4 = 12 marks]

11(*a*) Let $\{(X_\alpha, \tau_\alpha) : \alpha \in \Lambda\}$ be an indexed family of topological spaces, and for each $\alpha \in \Lambda$, let B_α be a basis for τ_α. Then prove that the collection B of all sets of the form $\prod_{\alpha \in \Lambda} B_\alpha$, where $B_\alpha = X_\alpha$ for all but a finite number of members $\beta_1, \beta_2, \ldots, \beta_n$ of Λ and $B_{\beta_i} \in B_{\beta_i}$ for each $i = 1, 2, \ldots, n$ is a basis for the product topology τ on $\prod_{\alpha \in \Lambda} X_\alpha$.

(*b*) Let $\{(X_\alpha, \tau_\alpha) : \alpha \in \Lambda\}$ be an indexed family of first countable spaces, and let $X = \prod_{\alpha \in \Lambda} X_\alpha$. Then (X, τ) is first countable if and only if τ_α is the trivial topology for all but a countable number of α.

[5 + 7 = 12 marks]

12 (*a*) Let (X, τ) and (Y, V) be topological spaces, let f be a continuous function that maps X onto Y, and let U be the quotient topology on Y induced by f. Prove that if f is open or closed, then $U = V$.

(*b*) Prove that the composition of two quotient maps is a quotient map.

(*c*) Suppose R has the usual topology. Define an equivalence relation ~ on R by $a \sim b$ provided there is an even integer k such that $a - b = k\pi$. Let D be the set of all equivalence classes, and let U be the quotient topology on D induced by the function that maps each member of R into the equivalence class that contains it. Describe the quotient space (D, U).

[4 + 4 + 4 = 12 marks]

UNIT I I I

13(a) When we say that a subset of a topological space is compact? Give one example.

(*b*) If X is an infinite set and τ is the discrete topology on X, then prove that (X, τ) is not compact.

(*c*) Prove that every totally bounded metric space is bounded.

[4 + 4 + 4 = 12 marks]

14(*a*) Prove that every countable compact space has the Bolzano-Weierstrass property.

(*b*) Prove that a metric space is compact if and only if it is closed and bounded.

[4 + 8 = 12 marks]

15(a) Prove that compactness is a topological property.

(*b*) Prove that product of two compact spaces is compact.

(*c*) Let (X, τ) be a compact space and let $f : X \to R$ be a continuous function. Then prove that there exist $c, d \in X$ such that for all $x \in X$, $f(c) \le f(x) \le f(d)$.

[4 + 4 + 4 = 12 marks]

First Semester M.Sc Degree Examination
Mathematics
Model Question Paper 2014 Admissions
MAT1C05: Differential Equations

Second Semester M.Sc Degree Examination
Mathematics
Model Question Paper 2014 Admissions

MAT2C06: Advanced Abstract Algebra

Time: Three Hours   Maximum : 60 Marks

Part A

Answer four questions from this part.

Each question carries 3 marks.

1. Prove that in a PID an irreducible is a prime.
2. Find a gcd of 8+6i and 5-15i in □$[i]$
3. Prove that $\sqrt{\sqrt[3]{2}-i}$ is an algebraic number.
4. Find the number of primitive 18th unity in GF(9).
5. Find the fixed field of $Q(\sqrt{2},\sqrt{3})$ over Q.
6. What is the order of $G\left(□\left(\sqrt[3]{2}\right) / □\right)$?

Part B

Answer **4** questions from this part without omitting any Unit. Each question carries **12** marks.

UNIT I

7. (*a*) Prove Guass Lemma.

   (*b*) Define UFD and show □$\left[\sqrt{-5}\right]$ is not a UFD.

8. (*a*) Prove the ascending chain condition for a PID.

   (*b)* Prove that an ideal $< p >$ in a PID is maximal iff p is an irreducible.

9. (a) Let p be an odd prime. Prove that $p = a^2+b^2$ for integers a and b in □ if and only if $p \equiv 1 (\mathrm{mod}\ 4)$

   (b) State and prove Knonecker's theorem.

UNIT I I

10. (a) Prove that the set of all constructible real numbers forms a subfield of $\mathbb{R}$.

(b) Prove that doubling the cube is impossible.

11. (a) Prove that a finite extension field *E* of field *F* is an algebraic extension of *F*.

(b) Prove that $\mathbb{Q}(\sqrt{2}) \cong \mathbb{Q}[x] / < x^2 - 2 >$

12. (*a*) If *E* is a finite field of characteristic p, then prove that then *E* contains exactly p^n elements for some positive integer *n*.

(*b*) Prove that complex zeros of polynomials with real coefficients occur in conjugate pairs.

UNIT I I I

13. State and prove isomorphism extension theorem.

14. (*a*) Define splitting field, give an example.

(*b*) Prove that A field *E*, where $F \leq E \leq \bar{F}$ is a splitting field over *F* iff every automorphism of $\bar{F}$ leaving F fixed maps E onto itself and thus induces an automorphism of F fixed.

15. (*a*) Prove that A field is perfect if every finite extension is a separable extension.

(*b*) State and prove primitive element theorem.

Second Semester M.Sc Degree Examination
Mathematics
Model Question Paper 2014 Admissions
MAT2C 07: Measure and Integration

Time: 3 hrs Max.Mark:60

Part A

Answer four questions form this part
Each questions carries 3 marks

1. Show that outer measures is translation invariant
2. If E_1 and E_2 are measurable sets, show that
   $m(E_1 \cup E_2) + m(E_1 \cap E_2) = m(E_1) + m(E_2)$
3. Let f be the function defined by f(0) = 0 and $f(x) = x \sin \frac{1}{x}$ if $x \neq 0$
   Find $D^+f(0)$ and D-f(0)
4. Show that the monotone convergence theorem need not hold for decreasing sequence of functions.
5. Prove that if ϑ is a signed measure such that $\vartheta << \mu$, then $\vartheta=0$
6. State Hahn Decomposition Theorem. Is it unique verify

Part B
Answer any four questions from this part without omitting any unit.
Each question carries 12 marks.
Unit – I

7. a) Show that the collection $\mathfrak{M}$ of measurable sets is a σ - algebra.
   b) Define a measurable function and show that the sum of two measurable functions is measurable.
8. State and prove fatous Lemma. Show that strict inequality may hold in fatous Lemma.
9. a) Construct a nonmeasurable set
   b) Prove linearity property of integrals for measurable functions.

Unit – II

10. a) State and prove Dominated convergence Theorem
    b) If is continuous on [a, b] then show that f is integrable and F(x) = $\int_a^x f(t)dt$ then prove that $F^1(x) = f(x)$ for almost all x in [a. b]

11. If f is Reimann Integrable and bounded on [a. b] show that f is integrable & R $\int_a^b f\,dx = \int_a^b f\,dx$ .Verify converse does not hold.

12. a) Define 1) Complete measure

2) σ – finite measure

b) if μ is a σ finite measure on a ring R, than it has a unique extension to the σ ring s(R)

Unit – III

13. a) Define measure space and measurable space, Give Example

b) State and prove monotone convergence theorem

14. a) Verify Holder's inequality

b) Verify Minkowski's inequality

15. Prove completeness of $L^p(\mu)$

.

Second Semester M.Sc Degree Examination
Mathematics
Model Question Paper 2014 Admissions
MAT2C08: T o p o l o g y

Time: Three Hours | Maximum : 60 Marks

Part A

Answer **four** *questions from this part.*
Each question carries **3** *marks.*

1. Prove that every subspace of a T_2 space is a T_2 space.
2. Give an example of a space that is normal but not regular.
3. Let (X, τ) be a topological space, and p be an object that does not belong to X, and let $Y = X \cup \{p\}$. Prove that $U = \{U \in P(Y) : U \in \tau$ or $Y - U$ is a closed compact subspace of $X\}$ is a topology on *Y.*
4. For each natural number number *n*, let (X_n, d_n) be a metric space, let $X = \prod_{n \in N} X_n$. Define

$$d(x, y) = \sum_{n=1}^{\infty} \frac{d_n(x_n, y_n)}{2^n}$$

for all $x = (x_1, x_2, \ldots, x_n), y = (y_1, y_2, \ldots, y_n) \in X$. Prove that d is a metric on *X.*

5. Let (X, τ) be a topological space and let $x_0 \in X$. Prove that $\square_p$ is an equivalence relation on $\Omega(X, x_0)$.
6. Explain the terms covering map and covering space.

Part B

Answer **four** *questions from this part without omitting any Unit.*
Each question carries **12** *marks.*

UNIT I

7. (*a*) Let (X, τ) be a topological space. Then prove that the following statements are equivalent.
   - (i) (X, τ) is a T_1 space.
   - (ii) For each $x \in X$, $\{x\})$ is closed.
   - (i) If *A* is any subset of *X,* then $A = \bigcap \{U \in \tau : A \subseteq U\}$.

(*b*) Prove the result: A T_1 space (*X*, τ) is regular if and only if for each member *p* of *X* and each neighborhood *U* of *p*, there is a neighborhood *V* of *p* such that $\bar{V} \subseteq U$.

[6 + 6 = 12 marks]

8. (*a*) Prove that every compact Hausdorff space is normal.

(*b*) Examine whether the Moore plane is normal or not.

(*c*) Let *C* be a closed subset of a normal space (*X*, τ). Then prove that (C, τ_C) is normal.

[4 + 4 + 4 = 12 marks]

9. (*a*) Prove that every second countable space is Lindelof.

(*b*) Prove that the set of dyadic numbers in $I = [0, 1]$ is dense in *I*.

(*c*) Prove that a topological space is completely regular if and only if it homeomorhphic to a subspace of a cube.

[4 + 4 + 4 = 12 marks]

UNIT I I

10. (*a*) Prove that closed subspace of a locally compact Hausdorff space is locally compact.

(*b*) Prove that every first countable Hausdorff space is a *k*-space.

(*c*) Let (*X*, τ) and (Y, U) be topological spaces and let *F* be a finite set of conintuous functions that map *X* into *Y*. Prove that *F* is equicontinuous.

[4 + 4 + 4 = 12 marks]

11. (*a*) Prove that every completely regular space has a compactification.

(*b*) Prove that the product of compact spaces is compact.

[5 + 7 = 12 marks]

12. State and prove Urysohn's metrization theorem.

[12 marks]

UNIT III

13. *(a)* Let (X, τ) and (Y, U) be topological spaces. Then prove that the function homotopy is an equivalence relation on *C(X, Y)*, the collection of continuous functions that map *X* onto *Y*.

(*b*) Let (X, τ) be a topological space, and let $x_0 \in X$. Furthermore, let $\alpha_1, \alpha_2, \beta_1, \beta_2 \in \Omega(X, x_0)$ and suppose $\alpha_1 \simeq_p \alpha_2$ and $\beta_1 \simeq_p \beta_2$. Then prove that $\alpha_1 * \beta_1 \simeq_p \alpha_2 * \beta_2$.

[6 + 6 = 12 marks]

14. (*a*) Let (X, τ) and (Y, U) be topological spaces, let $x_0 \in X$ and $y_0 \in Y$, and let $h:(X, x_0) \to (Y, y_0)$ be a map. Then prove that *h* induces a homeomorphism $h_*:\pi_1(X, x_0) \to \pi_1(Y, y_0)$. .

(*b*) Let (X, τ) , (Y, U) and (Z, V) be topological spaces, let $x_0 \in X$, $y_0 \in Y$, and $z_0 \in Z$, and let $h:(X, x_0) \to (Y, y_0)$ and $k:(Y, y_0) \to (Z, z_0)$ be maps. Then prove that $(k \circ h)_* = k_* \circ h_*$.

. [6 + 6 = 12 marks]

15. (a) Let $\alpha : I \to S^1$ be a path and let $x_0 \in R$ such that $p(x_0) = \alpha(0)$. Then prove that there is a unique path $\beta : I \to R$ such that $\beta(0) = x_0$ and $p \circ \beta = \alpha$.

(b) Describe the term covering space of a topological space.

. [10 + 2 = 12 marks]

Second Semester M.Sc Degree Examination
Mathematics
Model Question Paper 2014 Admissions
MAT2C09: Foundations of Complex Analysis

Time: 3hrs. Max.Mark:60

Part A
Answer four questions form this part
Each questions carries 3 marks

1. Derive Cauchy's estimate
2. F(z) = $\frac{1}{z(z\ 1)(z-2)}$ Give the Laurent

   Expansion of f(z) in different regions.
3. Find $\int_0^{\pi} \frac{d\theta}{z+Cos\theta}$ z >1,z ,a constant
4. Verify Hurwitz Theorem
5. Derive an application of the weierstrass factorization to sin πz
6. Define the Gamma function. Derive its functional Equation

Part B
Answer any four questions from this part without omitting any unit.
Each question carries 12 marks.

Unit – I

7. a) Let G be an open disk of u : G $\rightarrow$ R is harmonic then u has harmonic conjugate. Verify

   b) State and prove liouvilles theorem .Derive fundamental theorem of Algebra
8. a) Prove Maximum Modulus Theorem

   b) If γ:[0,1] → C is closed rectifiable curve and a is not in { γ} the prove $\int_{\gamma} \frac{dz}{z-a}$ is an integral multiple of 2π i
9. a) state an prove open mapping Theorem

   b) State an prove Goursat's Theorem

Unit – II

10. Derive Laurent series Development

11. a) State and prove Cauchy's Residue Theorem

    b) Show that $\int_{-\infty}^{\infty} \frac{x^2}{1+x^4}$ dx = $\frac{\pi}{\sqrt{2}}$

12. a) State and prove Schwarz's Lemma

    b) State and prove maximum modulus Theorem third version.

Unit – III

13. a) Show that C(G, Ω) is complete metric space

    b) Prove A set $F \subset$ C(G, Ω) is normal off its closure is compact

14. Prove Arzela – Ascolis Theorem

15. a) Prove Fundamental criterion for convergence a infinite product

    b) Prove Bohr – Mollerup Theorem

Second Semester M.Sc Degree Examination
Mathematics
Model Question Paper 2014 Admissions
MAT2C10: Partial Differential Equations

Third Semester M.Sc Degree Examination
Mathematics

Model Question Paper 2014 Admissions

MAT3C11: Number Theory

Third Semester M.Sc Degree Examination
Mathematics
Model Question Paper 2014 Admissions
MAT3C12: Functional Analysis

Time: Three Hours | Maximum : 60 Marks

Part A

Answer **four** *questions from this part.*
Each question carries **3** *marks.*

1 Let $X = K^3$. For $x = (x(1), x(2), x(3)) \in X$, let $\|x\| = \left[\left(|x(1)|^2 + |x(2)|^2\right)^{3/2} + |x(3)|^2\right]^{1/3}$. Show that $\|\cdot\|$ is a norm on K^3.

2 Define a Banach limit. Prove that the sequence $b = (1, 0, 0, 1, 0, 0, \ldots)$ is almost convergent, and $f(b) = \frac{1}{3}$ for every Banach limit *f*.

3 Give an example of a set of continuous functions from a metric space to a metric space that are bounded at each point without being uniformly bounded.

4 Show that a continuous map on a normed space is a closed map. Is the converse true? Justify your answer.

5 Let X be a complex inner product space and $A : X \to X$ be linear. Show that for $x, y \in X$,

$$4\langle A(x), y\rangle = \langle A(x+y), x+y\rangle - \langle A(x-y), x-y\rangle + i\langle A(x+iy), x+iy\rangle - i\langle A(x-iy), x-iy\rangle.$$

6 Let X be an inner product space and $E \subset X$ convex. Show that there exists atmost one best approximation from *E* to any $x \in X$.

Part B

Answer **four** *questions from this part without omitting any Unit.*
Each question carries **12** *marks.*

UNIT I

7 (*a*) Prove that vector addition and scalar multiplication are continuous functions on any normed space X and deduce that if E_1 is open subset and E_2 any subset of a normed space *X*, then $E_1 + E_2$ is open in *X*.

(*b*) Show that the norms $\| \|_1$, $\| \|_2$ and $\| \|_\infty$ on K^n are equivalent.

(*c*) State and prove Riesz Lemma.

[3 + 3 + 6 = 12 marks]

8(*a*) Let *X* and *Y* be normed spaces and $F: X \to Y$ be a linear map. Show that *F* is continuous on *X* if and only if $\|F(x)\| \le \alpha \|x\|$ $\forall x \in X$ and some $\alpha > 0$.

(*b*) Show that a linear map on a linear space *X* may be continuous with respect to some norm on *X*, but discontinuous with respect to another norm on *X*.

[6 + 6 = 12 marks]

9(*a*) Let *X* be a normed space over K, *Y* be a subspace over *X* and $g \in Y'$. Show that there is some $f \in X'$ such that $f_{|Y} = g$ and $\|f\| = \|g\|$.

(*b*) Show that a normed space can be emebedded as a dense subspace of a Banach space.

[7 + 5 = 12 marks]

UNIT I I

10 (*a*) State and prove uniform boundedness principle.

(*b*) Show that a subset *E* of a normed space *X* is bounded in *X* if and only if $f(E)$ is bounded in K for every $f \in X'$.

[7 + 5 = 12 marks]

11(*a*) Let *X* and *Y* be Banach spaces and $F: X \to Y$ be a closed linear map. Show that *F* is continuous.

(*b*) Let *X* and *Y* be normed spaces. If *Z* is a closed subspace of *X,* then show that the quotient map *Q* from *X* to *X/Z* is continuous and open.

[9 + 3 = 12 marks]

12 (*a*) State and prove open mapping theorem.

(*b*) Let *X* and *Y* be Banach spaces and let $F \in BL(X, Y)$ be bijective. Show that $F^{-1} \in BL(Y, X)$.

[9 + 3 = 12 marks]

UNIT I I I

13(b) Let $\langle\ ,\ \rangle$ be an inner product on a linear space *X*. Show that for all $x, y \in X$, $|\langle x, y\rangle|^2 \le \langle x, x\rangle\langle y, y\rangle$. Under what condition equality holds. Justify your claim.

(*b*) Show that an inner product $\langle\ ,\ \rangle$ on a linear space *X* induces a norm on *X*.

(*c*) Let X and Y be inner product spaces. Show that a linear map $F : X \to Y$ satisfies $\langle F(x), F(y)\rangle = \langle x, y\rangle \ \forall x, y \in X$ if and only if it satisfies $\|F(x)\| = \|x\| \ \forall x \in X$, where the norms on X and Y are induced by the respective inner products.

[6 + 3 + 3 = 12 marks]

14(*a*) Let $\{u_\alpha\}$ be an orthonormal set in a Hilbert space H. Show that $\{u_\alpha\}$ is an orthonormal basis for H if and only if $x \in H$ and $\langle x, u_\alpha\rangle = 0$ for all α, implies $x = 0$.

(*b*) Show that a non-zero Hilbert space H over K has a countable orthonormal basis if and only if H is separable.

[6 + 6 = 12 marks]

15(*a*) Let F be a subspace of X and $x \in X$. Then prove that $y \in F$ is a best approximation from F to x if and only if $x - y \perp F$ and in that case

$$\text{dist } (x, F) = \langle \qquad \rangle^{-}$$

(*b*) State and prove Riesz representation theorem.

(*c*) Show that the completeness assumption in the projection theorem cannot be omitted.

[3 + 6 + 3 = 12 marks]

Third Semester M.Sc Degree Examination
Mathematics
Model Question Paper 2014 Admissions
MAT3C13: Complex Function Theory

Time: 3hrs. Max.Mark:60

Part A
Answer four questions form this part
Each questions carries 3 marks

1. Define Modular function $\lambda(T)$. Show that $\lambda(T+1) = \frac{\lambda(T)}{\lambda(T)-1}$
2. Define Analytic continuation along a path
3. State and Prove Mean value Theorem for Harmonic functions
4. Define sub harmonic and super harmonic functions. State one result which hold for both harmonic and sub harmonic functions.
5. What in finite rank of an elliptic function? Define standard form of such function.
6. If f be an entire function of finite order. Show that f assumes each complex number with one possible exception.

Part B
Answer any four questions from this part without omitting any unit.
Each question carries 12 marks.
Unit – I

7. a) Define Riemann Zeta function and Find a relationship between the Zeta function and the Gamma function

   b) Prove Euler's Theorem
8. Define Canonical basis

   Prove Existence and uniqueness of such a basis.
9. a) Derive the differential Equation of $\rho(z)$

b) Prove Addition Theorem for ρ – function

Unit – II

10. Let K be a compact subset of the region G; then there are straight line segments $\gamma_1 \ldots\ldots \gamma_n$ in G-K such that for every function f in H(G)

$$f(z) = \sum_1^n \frac{1}{2\pi i} \int_{\gamma k} \frac{f(w)}{w-z} dw$$

for every z in K

11. a) State and prove Mittag – Leffler's Theorem

    b) Verify above with an example

12. State and prove Monodromy Theorem

Unit – III

13. Define Poisson kernel

    Derive four properties.

    b) If u in harmoric show that

    $f = u_x - iu_y$ is analytic

14. a) State and prove Harnack's Theorem

    b) If u: G → R is continuous function which has the MVP then show that u is harmonic

15. a) State and prove Jensen's Formula

    b) If f is an entire function of finite genus μ then f is of finite order $\lambda \le \mu + 1$. Verify

Third Semester M.Sc Degree Examination
Mathematics
Model Question Paper 2014 Admissions
MAT3C14: Advanced Real Analysis

Time: 3hrs. Max.Mark:60

Part A
Answer four questions form this part
Each questions carries 3 marks

1. 'A convergent series of continuous function may have a discontinuous sum' Justify.
2. $f(x) = x \quad o \le x < 2\pi$, Apply parseval's theorem to conclude that
   $\sum_1^\infty \frac{1}{n^2} = \pi^2/6$
3. Define 1) Fourier Series
   2) Fourier Coefficients
4. $f_n(x) = \frac{x}{1+nx^2}$ show that (f_n) converges uniformly to a function f and f_n^I converges to f^I if $x \ne o$
5. State the stone wierstrass Theorem
6. Define point wise bounded and uniform bounded for sequence of functions.

Part B
Answer any four questions from this part without omitting any unit.
Each question carries 12 marks.

Unit – I

7. a) Verify Cauchy criterion for uniform convergence of sequence of functions.
   b) Discuss weierstrass test for uniform convergence of functions.
8. a) $\propto$ is increasing on [a,b] Suppose (f_n) sequence in $R(\alpha)$ on [a,b] and suppose f_n converges to f uniformly on [a,b]. Show that f is in $R(\propto)$ on [a,b] and $\int_a^b f dd$ = Lim $\int_a^b fnd \propto$

b) Deduce term by term integration is possible for series of functions.

9. State and prove the stone weierstrass theorem

Unit – II

10. a) suppose $\sum_o^\infty c_n \quad x^n$ converges for $|x| < R$

Define f(x) = $\sum_o^\infty c_n \quad x^n$ in (-R,R)

Show that the above series converges uniformly and f is continuous and differentiable in (-R, R)

b) Deduce f^k(o) = k! c_k

11. a) Verify that the complex field is algebraically complete.

b) If f is continuous (with period 2π) and ϵ < o show that there is a trigonometric polynomial P such that $|P(x) – f(x)| < \epsilon$

12. a) state and prove parseval's theorem

b) Define Gamma function. Show that log Γ is convex on (o, ∞)

Unit – III

13. State and prove inverse function theorem

14. State and prove implicit function theorem

15. a) Suppose f maps a convex open set

$E \subset IR^n$ into IR^m, f is differentiable in E and there is M such that

$\| f^1 (x) \| \leq M$ for every x in E Then show that $|f(b) – f(a)| \leq M |b\text{-}a|$ fora, b in E.

b) f (x,y) = $\frac{xy}{x^2 + y^2}$ (x, y) ≠ o

= 0 at (o, o)

Prove $(D_1 f)$ (x,y) & $(D_2 f)$ (x,y) exist

at all points even though f is not continuous at o

Fourth Semester M.Sc Degree Examination
Mathematics
Model Question Paper 2014 Admissions
MAT4C15 : Operator Theory

Time: Three Hours | Maximum : 60 Marks

Part A

Answer **four** *questions from this part.*
Each question carries **3** *marks.*

1. Let X be a normed space over K and $k \in \sigma(A)$. Show that

   (i) $k^{-1} \in \sigma(A^{-1})$ if A is invertible.

   (ii) $k^2 \in \sigma(A^2)$.

2. Let X and Y be normed spaces and $F \in BL(X, Y)$. Define the transpose F' of F. Also show that $\|F'\| = \|F\|$.
3. Let X and Y be normed spaces and $F : X \to Y$. Show that F is compact if and only if for every bounded sequence (x_n) in X, $(F(x_n))$ has a convergent subsequence.
4. Let X be a reflexive normed space. Show that every closed subspace of X is reflexive.
5. Let H be a Hilbert space. $A \in BL(H)$ and F be a closed subspace of H. Then show that $A(F) \subset F$ if and only if $A^*(F^\perp) \subset F^\perp$.
6. Let H be a Hilbert space, $A \in BL(H)$. Show that A is normal if and only if $\|Ax\| = \|A^* x\|$ for all $x \in H$.

Part B

Answer **four** *questions from this part without omitting any Unit.*
Each question carries **12** *marks.*

UNIT I

7. (*a*) Let X be a normed space over K and $A \in BL(X)$ and $P(x)$ be a polynomial. Show that if $k \in \sigma(A)$, then $P(k)$ is in $\sigma(P(A))$.

   (*b*) Let X be a Banach space, $A \in BL(X)$ and $\|I - A\| < 1$. Show that A is invertible.

[6 + 6 = 12 marks]

8. (*a*) Let $1 \le p \le \infty$ and $\frac{1}{p} + \frac{1}{q} = 1$. Show that the dual of K^n with the norm $\| \ \|_p$ is linearly isometric to K^n with the norm $\| \ \|_q$.

(*b*) Let *X* be a normed space and X' is separable, prove that *X* is separable. Is the converse true? Justify your answer.

[6 + 6 = 12 marks]

9. (*a*) Let *X* be a normed and (x_n) be a sequence in *X*. Then prove that (x_n) is weak convergent in *X* if and only if

(i) (x_n) is a bounded sequence in *X* and

(ii) there is some $x \in X$ such that $x'(x_n) \to x'(x)$ for every x' in some subset X' whose span is dense in X'.

(*b*) Let *X* be a separable normed space. Then show that every bounded sequence in X' has a weak* convergent subseqeunce.

[8 + 4 = 12 marks]

UNIT I I

10. (*a*) Let *X* be a reflexive normed space. Show that every closed subspace of *X* is reflexive.

(*b*) Examine the reflexivity of ℓ^p, $1 \le p \le \infty$. [6 + 6 = 12 marks]

11. (*a*) Let *X* be a normed space, *Y* be a Banach space, $F_n \in CL(X, Y)$, $F \in BL(X, Y)$ and $\|F_n - F\| \to 0$. Prove that $F \in CL(X, Y)$.

(*b*) Let *X* and *Y* be a normed spaces and $F \in BL(X, Y)$. If $F \in CL(X, Y)$, then prove that $F' \in CL(Y', X')$. Also show that the converse holds if *Y* is a Banach space.

[4 + 8 = 12 marks]

12. (*a*) If *X* is an infinite dimensional normed space and $A \in CL(X)$. Show that $0 \in \sigma_a(A)$.

(*b*) Show that the eigen spectrum and the spectrum of a compact operator on a normed space *X* are countable and have zero as the only possible limit point.

[6 + 6 = 12 marks]

UNIT III

13. (*a*) If H is a Hilbert space and $A \in BL(H)$, prove that A has a unique adjoint in $BL(H)$.

 (*b*) Can completeness assumption omitted in the (*a*) part? Explain.

 (*c*) Let H be a Hilbert space and $A \in BL(H)$ be self adjoint. Show that $A^2 \geq 0$ and $A \leq \|A\| \, I$, where I is the identity operator on H.

[5 + 3 + 4 = 12 marks]

14. (*a*) Define the numerical range of a bounded operator A on a Hilbert space H and show that it is bounded.

 (*b*) Let H be a Hilbert space and $A \in BL(H)$ be compact. Show that A^* is compact.

[6 + 6 = 12 marks]

15. (*a*) Let H be a nonzero Hilbert space and A be a self adjoint compact operator on H. Show that $\|A\|$ or $-\|A\|$ is an eigen value of A.

 (*b*) Show that every Hilbert Schmidt operator on a separate Hilbert space is compact.

[6 + 6 = 12 marks]

Fourth Semester M.Sc Degree Examination
Mathematics
Model Question Paper 2014 Admissions
MAT4C16: Differential Geometry

Time: 3 Hours Max. Marks: 60

PART – A

Answer **any four** questions. **Each** question carries **3** marks.

1. Find and sketch the gradient vector field of the function f defined by $f(x_1, x_2) = x_1^2 + x_2^2$, $x_1, x_2 \in \mathbb{R}$.
2. Show that the special linear group SL(2) is a 3-surface in $\mathbb{R}^4$.
3. Show that the spherical image of an n-surface S with orientation N is the reflection through the origin of the spherical image of S with orientation – N.
4. Find the Weingarten map of the sphere $x_1^2 + x_2^2 + x_3^2 = 2$ oriented by the outward unit normal vector field.
5. Find the curvature of $(x_1 - a)^2 + (x_2 - b)^2 = r^2$, $r > 0$ oriented by the outward normal vector field.
6. Find the length of the parametrized curve $\alpha : [-1, 1] \longrightarrow \mathbb{R}^3$ defined by $\alpha(t) = (\cos 3t, \sin 3t, 4t)$.

PART – B

Answer **any four** questions without omitting any unit. **Each** question carries 12 marks.

UNIT – I

7. a) Sketch typical level curves and the graph of the function $f : \mathbb{R}^2 \longrightarrow \mathbb{R}$ defined by $f(x_1, x_2) = -x_1^2 + x_2^2$.

   b) Let≠ be the vector field on $\mathbb{R}^2 : \mathbb{X}(p) = (p, X(p))$ where $X(x_1, x_2) = (-x_2, x_1)$. Find the integral curve of $\mathbb{X}$ through $p = (1, 0)$.

c) Let U be an open set in $\mathbb{R}^{n+1}$ and let $f : U \to \mathbb{R}$ be smooth Let $p \in U$ be a regular point of f, and let $c = f(p)$. Prove that the set of all vectors tangent to $f^{-1}(c)$ at p is equal to $[\nabla f(p)]^{1}$.

8. a) State and prove Lagrange multiplier theorem.

b) Let $a, b, c \in \mathbb{R}$ be such that $ac - b^2 > 0$. Show that the maximum and minimum values of the function of $(x_1, x_2) = x_1^2 + x_2^2$ on the ellipse $a x_1^2 + 2bx_1, x_2 + c x_2^2 = 1$ are of the form $\frac{1}{\lambda_1}$ and $\frac{1}{\lambda_2}$ where λ_1 and λ_2 are the eigen values of the matrix $\begin{pmatrix} a & b \\ b & c \end{pmatrix}$.

9. a) Define an n-surface in $\mathbb{R}^{n+1}$. Sketch the cylinders $g^{-1}(1)$ over the n–sphere : $g(x_1, \ldots, x_{n+1}) = x_1^2 + \ldots + x_n^2$, for n = 1, 2.

b) Show that if S is a connected n - surface in $\mathbb{R}^{n+1}$ and $g : S \to \mathbb{R}$ is smooth and taken on only the values +1 and –1, then g is constant.

c) Prove that each connected n-surface in $\mathbb{R}^{n+1}$ has exactly two orientations.

UNIT – II

10. a) Prove that for a compact, connected oriented n-surface S in $\mathbb{R}^{n+1}$ with $S = f^{-1}(c)$, $f : \mathbb{R}^{n+1} \to \mathbb{R}$ is a smooth function with $\nabla f(p) \neq 0$ for all p in S, the Gauss map $N : S \to S^n$ is onto.

b) Find the geodesics on the circular cylinder $x_1^2 + x_2^2 = 1$ in $\mathbb{R}^3$.

11. a) Let S be an n-surface in $\mathbb{R}^{n+1}$, let $\alpha : I \to S$ be a parametrized curve in S, Let $t_0 \in I$ and let $\mathbb{V} \in S_{\alpha(t_0)}$. Prove that there exists a unique vector field $\mathbb{V}$ tangent to S along α, which is parallel and $\mathbb{V}(t_0) = \mathbb{V}$.

b) Let $\alpha : [0, \pi] \to S^2$ be the half great circle in S^2, running from the north pole p = (0, 0, 1) to the south pole q = (0, 0, –1) defined by $\alpha(t) = (\sin t, 0, \cos t)$. Show that for $\mathbb{V} = (p, v_1, v_2, 0)$ in S_p^2, $P_{\alpha}(\mathbb{V}) = (q, -v_1, v_2, 0)$.

12. a) Let S be an n-surface in $\mathbb{R}^{n+1}$ oriented by the unit normal vector field $\mathbb{N}$. Let $p \in S$ and $\mathbb{V} \in S_p$. Prove that every parametrized curve $\alpha : I \to S$ with $\dot{\alpha}(t_0) = \mathbb{V}$ for some $t_0 \in I$, $\ddot{\alpha}(t_0) . \mathbb{N}(p) = L_p(\mathbb{V}).\mathbb{V}$.

b) Let $\alpha(t) = (x(t), y(t)), t \in I$ be a local parametrization of the oriented plane curve C. Show that $K \circ \alpha = (x'y'' - y'x'')/(x'^2 + y'^2)^{3/2}$.

c) Show that local parametrizations of plane curves are unique up to reparametrization.

UNIT – III

13. a) Let C be a connected oriented plane curve and let $\beta : I \to C$ be a unit speed global parametrization of C. Then prove that either β is one to one or periodic.

b) Let η be the 1-form on $\mathbb{R}^2 - \{0\}$ defined by $\eta = -\frac{x_2}{x_1^2 + x_2^2} dx_1 + \frac{x_1}{x_1^2 + x_2^2} dx_2$. Then prove that for $\alpha : [a, b] \to \mathbb{R}^2 - \{0\}$ any closed piece wise smooth parametrized curve is $\mathbb{R}^2 - \{0\}$, $\int_\alpha \eta = 2\pi k$ for some integer k.

14. a) Let S be an n-surface in $\mathbb{R}^{n+1}$ and let V be a unit vector in S_p, $p \in S$. Prove that there exists an open set $V \subset \mathbb{R}^{n+1}$ containing p such that $S \cap N(V) \cap V$ is a plane curve. Further prove that the curvature at p of this curve (suitably oriented) is equal to the normal curvature $k(V)$.

b) Find the Gaussian curvature of the ellipsoid $\frac{x_1^2}{a^2} + \frac{x_2^2}{b^2} + \frac{x_3^2}{c^2} = 1$ (a, b and c all $\neq 0$) oriented by its outward normal.

15. a) Define a parametrized n-surface in $\mathbb{R}^{n+k}$ $(k \geq 0)$. Obtain a torus as a parametrized surface in $\mathbb{R}^3$.

b) Let $\alpha : U \to \mathbb{R}^{n+1}$ be a parametrized n-surface in $\mathbb{R}^{n+1}$ and let $p \in U$. Prove that there exists an open set $U_1 \subset U$ about p such that $\alpha(U_1)$ is an n-surface in $\mathbb{R}^{n+1}$.

c) Give an example of a 2-surface in $\mathbb{R}^4$.

Third Semester M.Sc Degree Examination
Mathematics
Model Question Paper 2014 Admissions
MAT 3 E01: Elective 1: Graph Theory

Time : 3 Hours Max. Marks: 60

PART A

I. *Answer any **4** questions. **Each** question carries **3** marks*

1. Prove that there exists no plane Hamiltonian graph with more than one face having exactly one face of degree 7 and all other face of degree 5 and 8.
2. Find the chromatic polynomial of a four cycle.
3. Prove that $\sum_{v\in V} d^-(v) = \varepsilon = \sum_{v\in V} d^+(v)$.
4. Prove that $r(3,3)=6$.
5. Prove that Petersen graph is 4- edge chromatic.
6. Show that a tree has at most one perfect matching.

PART B

*Answer any **4** questions without omitting any **Unit**. Each question carries **12** marks*

Unit I

II. a. Define the independence number and covering number of a graph and prove that the sum of the

independence number and covering number is the number of vertices.

b. For any two positive integers k,l, prove that $r(k,l) \le r(k,l-1) + r(k-1,l)$ and the inequality

is strict if $r(k,l-1)$, $r(k-1,l)$.

III. a. If a simple graph G contains no K_{m+1}, prove that G is degree majorised by some complete

m- partite graph H. Further if G has the same degree sequence of H, prove that $G \cong H$.

b. Prove that in any critical graph no vertex cut is a clique and hence deduce that every critical graph is a block.

IV. a. If G is a connected simple graph such that it is neither an odd cycle nor a complete graph then prove that $\chi \leq \Delta$

b. If G is a simple graph, prove that $\pi_k(G) = \pi_k(G-e) + \pi_k(G.e)$.

Unit II

V. a. Prove that for a bipartite graph $\chi'(G) = \Delta$.

b. If G is a simple graph, prove that $\chi'(G)$ is either Δ or $\Delta + 1$.

VI. a. Derive the Euler's formula for a connected graph and hence deduce that $K_{3,3} - e$ is planar.

b. Prove that every planar graph is 5 vertex colourable.

VII. State and Prove Kuratowski's theorem.

Unit III

VIII. a. Prove that a digraph D contains a directed path of length $\chi - 1$.

b. Define a tournament. Draw the tournaments of three vertices.

c. Prove that every tournament has a directed Hamilton path.

IX. a. Prove that a loopless digraph D has an independent set such that every vertices of D not in the set are reachable by a directed path of length at most 2.

b. Prove that each vertex of a disconnected tournament of at least 3 vertices contained in directed $k-$cycle with $3 \leq k \leq v$.

X. a. Let f be a flow on a network $N = (V, A)$ and let f have value d. If $A(X, \bar{X})$ is a cut in N then prove that $d = f(X, \bar{X}) - f(\bar{X}, X)$ and $d \leq c(X, \bar{X})$

. b. State and prove max-flow, min-cut theorem.

Third Semester M.Sc Degree Examination
Mathematics
Model Question Paper 2014 Admissions
MAT 3 E02: Probability
Theory

Time : 3Hrs **Maximum : 60 marks**

Part A

(Answer any 4 questions. Each question carries 3 marks)

1. Prove that a σ- field is a monotone field and conversely.
2. Define a Random Variable. Give an example.
3. State and prove the Borel-Cantelli Lemma.
4. Define the distribution function of a Random Variable. Prove that the distribution function of a Random Variable is non-decreasing and right continuous.
5. Define the Characteristic function of a Random Variable and prove that it is uniformly continuous on the real line.
6. Give the Kolmogorov definition of Expectation. Explain using an example.

Part B

(Answer 4 questions not omitting any unit.)

Unit I

7. a) For a given algebra A, prove that the monotone class generated by A is the same as the σ- field generated by A.
   b) Write a brief note about the Borel sets in the real line.

8. a) State and Prove the continuity theorem of measure.
   b) Define a discrete Random Variable. Explain using an example.

9. a) For $n \in N$, let A_n denote the interior of the circle of unit radius with centre at $((-1)^n/n, 0)$. Find lim sup A_n, lim inf A_n. Also find lim A_n if it exists.
   b) Let $A_n = \{ (x, y) \in R^2 : 0 \le x \le n, 0 \le y < 1/n \}$, $n \in N$. Show that $\{ A_n \}$ is not monotone, but lim A_n exists . Also find lim A_n .

Unit II

10. a) Define the generating function of a real sequence $\{a_n\}$. If P(s) is the generating function of the Random Variable X, find the generating functions of X + 1 and 2X.
    b) Prove that the product of generating functions of two numerical sequences $\{a_k\}$ and $\{b_k\}$ is the generating function of the convolution of $\{a_k\}$ and $\{b_k\}$.

11. a) Define a degenerate distribution function and give an example.
    b) State and Prove Helly – Bray theorem.

12. a) State and Prove the inversion theorem for characteristic functions.
b) Explain the continuity theorem for characteristic functions.

Unit III

13. a) Show that a sequence $\{F_n\}$ of distributions may not converge at all.
b) State and Prove Helly's selection principle.

14. a) Let $\{f_n\}$ be a sequence of measurable functions converging to f in measure. Prove that there exists a subsequence $\{f_{nk}\}$ converging to f a.e.
b) Prove that every real valued real measurable function can be approximated by an increasing sequence of simple measurable functions.

15. a) State and Prove dominated convergence theorem.
b) Show by an example that for a Random Variable X, the expectation E(X) may not exist, but the Cauchy principal value may exist.

Fourth Semester M.Sc Degree Examination
Mathematics
Model Question Paper 2014 Admissions
MAT4E03: Calculus of Variations

Time: 3 hrs Max.Marks: 60

Part A

Answer four questions form this part
Each questions carries 3 marks

Answer any four questions. Each question carries 3 marks

1. Find the extremals of the function $\int_a^b (y^2 + y'^2 + 2ye^x)$dx

2. Find the extremals of the functional $\int_{x_0}^{x_1}(2yz \quad 2y^2 + y'^2 - z'^2)$dx

3. Find the transversality condition for functional of the form

   J[y]= $\int_{x_0}^{x_1} f(x, y)\sqrt{1 + y'^2}$dx Interpret the conditions geometrically.

4. Derive the canonical Euler equations of the functional $\int_{x_0}^{x_1} Py'^2 + Qy^2)$dx,

   where P and Q are functions of x.

5. Write the Hamilton-Jacoby equation for the functional J[y]= $\int_{x_0}^{x_1} y'^2$dx

6. Calculate the second variation of the functional $e^{J[y]}$, where J[y] is a twice

   differentiable functional. **(4 x 3 = 12)**

Part B
Answer any four questions from this part without omitting any unit.
Each question carries 12 marks.

Answer **any Four** questions without omitting any **Unit. Each** question carries 12 marks

UNIT - 1

7. a) Define the variation of a functional and show that the variation of a

   differentiable functional is unique.

b) Let J[y] $\int_a^b F(x, y, y')$dx be defined on the set of functions y(x) which have continuous partial derivatives in [a, b] and with y(a) =A, y(b)=B. Show that if y(x) is an entremal of J(y), then $F_y - \frac{d}{dx}F_{y'} = 0$

8. a) Describe the concept of the variational derivative of the functional J[y] = $\int_a^b F(x, y, y')$dx, y(a)=A, y(b)=B and derive an expression for it.

   b) Establish the invariance of Euler's equation under a transformation of the form x=x(u, v), y=y(u, v) with det $\begin{pmatrix} x_u & x_v \\ y_u & y_v \end{pmatrix} \neq 0$

   c) Find the extremals of the functional J(r) = $\int_{\Phi_0}^{\Phi_1} \sqrt{r^2 + r'^2}\, d\Phi$ where r = r(Φ)

9. a) Let J[y] = $\int_a^b F(x, y, y')$dx, where y(x) satisfy the conditions. y(a) = A, y(b)=B, K[y]= $\int_a^b G(x, y, y')$ dx= l; and let J[y] have an extremum for y=y(x). Prove that if y=y(x) is not an extremal or K[y], then there exists a constant λ such that y = y(x) is an extremal of $\int_a^b (F + \lambda G)$dx

   b) Find the curve in the xy-plane of fixed length / which passes through the points (-a, 0) and (a, 0) and for which the area bounded by the curve and the x-axis is maximum.

UNIT – II

10. a) Derive the basic formula for the general variation of the functional J[y_1, ... y_n]= $\int_{x_0}^{x_1} F(x, y_1, \ldots, y_n, y'_1, \ldots, y'_n)dx$

    b) Find the curves for which the functional J[y] = $\int_0^{x_1} \frac{\sqrt{1+y'^2}}{y}$ dx, y(0)=0 can have an extremum if the point (x_1, y_1) can vary along the line y= x-5

11. a) Derive the canonical system of Euler equations for the functional

$J[y_1, \ldots\ldots, y_n] = \int_a^b F(x, y_1, \ldots\ldots, y_n, y'_1, \ldots\ldots, y'_n)dx$

b) Use the canonical Euler equations to find the extremals of the functional $\int \sqrt{x^2 + y^2}\sqrt{1 + y'^2}dx$

12. a) State and prove Noether's theorem on invariance of the functional $J[y] = \int_a^b F(x, y, y')dx$, Under a family of transformations.

b) State the principle of least action and deduce the law of conservation of momentum.

UNIT – III

13. a) Prove that a necessary condition for the functional J[y] to have a minimum for $y = \hat{y}$ is that $\delta^2 J[y] \geq 0$ and all admissible h.

b) With usual notations, show that the second variation of the functional $J[y] = \int_a^b F(x, y, y')dx$ defined for curves y=y(x) with fixed end points y(a)=A, y(b)=B can be expressed as $\delta^2 J[h] = \int_a^b (P h'^2 + Q h^2)dx$

14. If the quadratic functional $\int_a^b (P h'^2 + Q h^2)dx$, where $P(x) > 0 (a \leq x \leq b)$, is positive definite for all h(x) such that h(a) = h(b)=0, then prove that the interval [a, b] contains no points conjugate to a .

15. a) State and prove a set of sufficient conditions for the functional $J[y] = \int_a^b F(x, y, y')dx$, y(a) = A, y(b) = B to have a weak minimum for the curve y = y(x)

b) State (without proof) three necessary conditions for the functional $J[y] = \int_a^b F(x, y, y')dx$, y(a) = A, y(b) = B to have a weak extremum for the curve y=y(x)

(4 x 12 = 48)

Fourth Semester M.Sc Degree Examination
Mathematics
Model Question Paper 2014 Admissions
MAT4E04: Commutative Algebra

Time: 3hrs. Max.Mark:60

Part A
Answer four questions form this part

Each questions carries 3 marks

1 Show that any nonzero ring has at least one maximal ideal

2 prove, M is finitely generated A-module iff M is isomorphic with a quotient of A^n for some n

3 M be an A-module .show that M = 0 iff $M_m = 0$ iff for all maximal ideals of m of A

4 If $\alpha = r(\alpha)$ show that α has no prime ideals

5 give an example which satisfies acc but not dcc . verify.

6 Verify whether the ring of power series in z with a positive radius of convergence is Noethrian

Part B

Answer any four questions from this part without omitting any unit.

Each question carries 12 marks.

Unit – I

7 a) Prove that the set ή of all nilpotent elements in ring is an ideal and A / ή has no nilpotent element not equal to 0

b) Prove that the nilpotent radical of A is the intersection of all the prime ideals of A.

8 a) let M be finitely generated A – module , let α be an ideal of A Φ be an A-module endomorphism of M such that Φ(M) is a subset of α(M), then show that Φ satisfies an equation of the form $\Phi^n + a_1\Phi^{n-1} +\ldots\ldots..+a_n = 0$ where a_i is in α

b) Prove Nakayamas lemma

9) a) let $M^1 \rightarrow M \rightarrow M^{11} \rightarrow 0$ be a sequence of A-module and homeomorphisms .State and prove the necessary and sufficient condition that the above sequence is exact .

b) In the ring A[x] , Is the Jacobson radical is equal to the nilradical . verify

Unit – II

10) a) g A$\to$ B be a ring homomorphism such that g(s) is a unit in B for every s in S. then prove that there is a unique ring homomorphism h : $s^{-1}A \to B$ such that $g = h \circ f$

B) Φ: M$\to$N be an A-module homomorphism ,then show that Φ is surjective iff

$\Phi_m: M_m \to N_m$ is surjective for each maximal ideal m

11) a) Let M be a finitely generated A-module ,S multiplicative closed subset of A. Show that $s^{-1}(A_{nn}(M)) = A_{nn}(s^{-1}M)$

b) Let α be an ideal of a ring A and let S = 1 + α , show that $s^{-1}\alpha$ is contained in the Jacobson radical of $s^{-1}A$

12) a) State and prove first Uniqueness theorem

b) Let K be a decomposable ideal in a ring A and Q be a maximal element of the set od ideals (K : x) where x is in A but x is not in K .show that Q is a prime ideal belonging to K

Unit – III

13) a) state and prove going up theorem and going down theorem

b) Let A be a subset of B be rings, B integral over A show that the Jacobson radical of A is the contraction radical of the Jacobson radical of B

14) a) Prove, M is a Noetherian A-module iff evey sub module of M is finitely generated

b) Suppose M has a composition series of length n .Show that every composition series of M has length n ,and every chain in M can be extended to a composition series .

15) a) State and prove Hilberts Basis theorem

b) let K be a field ,E finitely generated k-algebra . If E is a field then show that it is a finite algebraic extension of K

Fourth Semester M.Sc Degree Examination
Mathematics
Model Question Paper 2014 Admissions
MAT4E05: Fourier and Wavelet Analysis

Time: 3hrs. Max.Mark:60

Part A

Answer four questions form this part

Each questions carries 3 marks

1 Define up sampling and down sampling operator with examples

2 Explain the term filter bank . under what condition perfect reconstruction possible in the filter bank

3 Define convolution of two vectors in $l^1(z)$ and $l^2(z)$

4 Show that forier transformation is 1-1 and ont in $l^2(z)$

5 If f is in $l^1(R)$, show that $|\int f(x)dx| \leq |f|$

6 Define circulant matrix and give example

Part B

Answer any four questions from this part without omitting any unit.

Each question carries 12 marks.

Unit – I

7 a)Show that(z*)^(n) = z^(n+M) where N = 2M and z is in $l^2(z_N)$

b) State and prove the necessary and sufficient condition for the existence of first stage wavelet basis for $l^2(z_N)$

8 a) Explain first stage shannon basis ans first stage real shannon basis and verify that it is wavelet basis

b)N = 2M , u is in $l^2(z_N)$ is such that $(R_{2k}\, v)$ is an orthonormal set of M elements .Form a first stage wavelet basis by using these vectors

9 a)N=2M, z is in $l^2(z_N)$ and x ,y and w are in l $l^2(z_{N/2})$) show that

D(z)∗w = D(z∗ U(w) and U(x) ∗ U(y) = U (x ∗ y)

b) Construct a p th stage wavelet basis from a pth stage filter sequence

Unit II

10) a) Define $L^1[-\Pi, \Pi)$ and $L^2[-\Pi, \Pi)$ .Verify relation between these two spaces

b) construct an orthonormal set in $L^2[-\Pi, \Pi)$

11) a) let $T : L^2[-\Pi, \Pi) \rightarrow L^2[-\Pi, \Pi)$ is bounded translation invariant linear transformation , for every m in Z show that there is λ_m in C such that $T(e^{im\theta}) = \lambda_m e^{im\theta}$

b) z in $l^2(z)$ and w in $l^1(z)$ show that $z * w$ lies in $l^2(z)$ and $\| z * w\| \leq \| z \| \| w\|_1$

12) a) z in $l^2(z)$ and v and w are in $l^1(z)$ show that $(z * w)$ ^ $(\theta) = (z)$ ^ $(\theta)(w)$ ^ (θ)

b) Define translation invariant operator on $l^2(z)$. if T is such operator show that T is convolution operator.

Unit III

13) If f is in $l^1(R)$ and $(g_t)_{t>0}$ is approximate identity show that lim $(g_t * f)(x) = f(x)$ if x is Lebesegue point of f

14) a)Define Gaussian function G and show that $G\wedge = \sqrt{2\Pi}G$
b) g is in $l^1(R)$ and $t > 0$,prove (g) ^ $(t\xi) = (g_t)$ ^ (ξ)

15) a) State and prove Fourier inversion theorem on $l^1(R)$

b)Verify the uniqness of Fourier transform in on $l^1(R)$

Fourth Semester M.Sc Degree Examination
Mathematics
Model Question Paper 2014 Admissions
MAT4E06: Operations Research

Time: 3hrs. Max.Mark:60

Part A

Answer four questions form this part

Each questions carries 3 marks

1) Define state transition matrix and explain how they are construct

2) Explain Hurturiz criterion

3) Define hazard rate. Commend on the hazard rate of exponential rate distribution

4) what do you mean by economic order quantity

5) describe axiomatic approach to information

6) A bag contains 100 white balls 50 black balls and 50 blue balls. Another bag contains 80 white balls and 40 blue balls .Determine the expected amount of information associated with experiment of drawing a ball from each bag and predicting its colour

Part B

Answer any four questions from this part without omitting any unit.

Each question carries 12 marks.

Unit – I

7) Consider the 3 Markov chain associated with the probability matrix

$$\begin{pmatrix} 0 & \frac{1}{3} & \frac{2}{3} \\ \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & 0 \end{pmatrix}$$

Prove the chain is irreducible .Find the limit distribution

8. A motor parts dealer find the cost of a particular item in a stock for a week is Rs 30and cost of a unit shortage is Rs 50. The probability distribution of weekly sales (in '000 items) as follows.

Weekly sales (000)	:	0	1	2	3	4	5	6
Probability	:	.10	.10	.20	.20	.20	.15	.05

How many units per week should the dealer order .Also find the expected Value of perfect information.

9 A tourist cab owner has 25 taxis in operation. He keeps three drivers as reserve to attend to calls, in case the scheduled driver reports sick. The probability distribution of sick drivers is as below : -

Number Sick :	0	1	2	3	4	5
Probability :	.20	.25	.20	.15	.12	.08

Use Monte Carlo method to estimate the utilization of reserve drivers and the probability that atleast one taxi will be at the road due to non-availability of a driver

UNIT-II

10 The failure time of a component is exponentially distributed with mean life equal to 35() hours. The design is modified so that the mean life is increased to 400 hours. What is the increase in reliability if in both cases the component is tested for 800 hours. Also find mean time between failures in both cases. _

11. Derive the Economic lot size formula with finite rate of replenishment and shortages are allowed. (You may make necessary other assumptions).

12 .Show that for a probabilistic inventory model with instantaneous demand and

no setup cost the optimum stoçk level Q can be obtained by the relationship .

$F(Q^0 -1) \leq C_2 / C_2 + C_1 \leq F(Q^0)$

UNIT-III

13) Define entropy of the probability distribution. Let $p_1, p_2, p_3, \ldots\ldots\ldots\ldots\ldots p_M$ be the probability distribution of a random variable, the show that $H(p_1, p_2, p_3, \ldots\ldots\ldots\ldots\ldots p_M) \leq \log M$ with equality iff $p_i = M^{-1}$ for all i

14) If the probability distribution $\mathcal{P}(p_1, p_2, p_3, \ldots\ldots\ldots\ldots\ldots, p_i \geq 0$ for all I and $\sum p_i = 1$is such that $\sum p_i \log i < \infty$,show that $H(\mathcal{P}) = -\sum p_i \log p_i < \infty$

15) A source without memory has five characters with the following probabilities of transmission

A	B	C	D	E
.30	.25	.20	.10	.15

Derive the Shannon –Fano encoding procedure to obtain uniquely decodable code to the above message ensemble. what is the average length ,efficiency and redundancy of the code that you obtain

First Semester M.Sc Degree Examination
Mathematics
Model Question Paper 2014 Admissions
MAT1C05: Differential Equations

Time: Three Hours Maximum : 60 Marks

Part A

Answer **four** *questions from this part.*
Each question carries **3** *marks.*

1. Define F (a,b,c,x).Show that, $F(a,b,c,x) = \frac{ab}{c} F(a+1, b+1, c+1, x)$

2. Determine the nature of the point x = 0 for the differential equation y'' + (sin x)y = 0

3. Obtain the recursion formula: $(n+1)p_{n+1}(x) = (2n+1)p_n(x)\; n\; p_{n-1}(x)$

4. Prove that the positive roots of $J_p(x)$ and $J_{p+1}(x)$ occur alternately, in the sense that between each pair of consecutive positive zeros of either there is exactly one zero of the other.

5 Show that (0,0) is an asymptotically stable critical point for the system

$$\frac{dx}{dt} = -3x^3 - y \;;\; \frac{dy}{dt} = x^5 - 2y^3$$

6. Define Lipschitz condition. Show that $f(x,y) = x$ satisfies the Lipschitz condition on any strip $x \leq b \,;\, -\infty < y < \infty$

PART-B

Answer **four** *questions from this part without omitting any Unit.*
Each question carries **12** *marks.*

UNIT I

7. a) Show that the series $y = 1 - \frac{x^2}{2^2} + \frac{x^4}{2^2.4^2} - \frac{x^6}{2^2.4^2.6^2} + \cdots$ converges for all x

and verify that it is a solution of the equation $xy'' + y' + xy = 0$

b) Show that $\tan x = x + \frac{1}{3}x^3 + \frac{2}{15}x^5 + \cdots$ by solving $y' = 1 + y^2, y(0) = 0$

in two ways.

8 a) Find the general solution of the equation $4x^2y'' - 8x^2y' = (4x^2 + 1)y = 0$

b) Find the general solution of the equation

$(1 - e^x)y'' + \frac{1}{2}y' + e^x y = 0,$ near the singular point $x = 0$

9 a) Find two linearly independent solution of the Chebyshev equation

$(1 - x^2)y'' - xy' + p^2y = 0)$,where p is a

constant. b) Prove that Hermite polynomials are orthogonal.

UNIT II

10. a) Solve the Legendre equation $(1 - x^2)y'' - 2xy' + p(p + 1)y = 0$

b) Show that i) $2J_p'(x) = J_{p-1}(x) - J_{p+1}(x)$

ii) $\frac{2p}{x}J_p(x) = J_{p-1}(x) + J_{p+1}(x)$

11 a)If $P_m(x)$ and $P_n(x)$ are Legendre Polynomials, Prove that

$$\int_{-1}^{1} P_m(x)P_n(x)\,dx = \begin{cases} 0 & if\ m \neq n \\ \frac{2}{2n+1} & if\ m = n \end{cases}$$

b) Prove that $J_3(x) = \sqrt{\frac{2}{\pi x}}\,sinx$

12. a) Find the general solution of the following system of equations

$\frac{dx}{dt} = x + y - 5t + 2$, $\frac{dy}{dt} = 4x - 2y - 8t - 8$

b) Find the general solution of the following system of equations

$$\frac{dx}{dt} = 4x - 2y, \quad \frac{dy}{dt} = 5x + 2y$$

UN
IT
III

13. a) Find the critical points, the differential equation of the path and solve the differential equation to find the path for the system $\frac{dx}{dt} = y(x^2 + 1), \frac{dy}{dt} = 2xy^2$

b) State Liapunov stability theorem. Show that the function $E(x,y) = ax^2 + bxy + cy^2$ is positive definite if and only if $a > 0$ and $b^2 - 4ac < 0$ and negative definite if and only if $a < 0$ and $b^2 - 4ac < 0$.

14. a) Verify that (0,0) is a simple critical point for the system

$$\frac{dx}{dt} = x - y - 3x^2y, \quad \frac{dy}{dt} = -2x - 4y + y\sin x$$

b) Determine the nature and stability properties of the critical point (0,0) for the system

$$\frac{dx}{dt} = -4x - y, \quad \frac{dy}{dt} = x - 2y$$

15. State and Prove Picard's Theorem

(4) **Second Semester M.Sc Degree Examination**

Mathematics

Model Question Paper 2014 Admissions

MAT2C10: PARTIAL DIFFERENTIAL EQUATIONS

Time 3 hours | Maximum Marks 60

(PART A)

Answer any four questions from this part.
Maximum Weightage from this part is 12.

1. Eliminate the arbitrary function F from $F(x-z, y-z) = 0$ and find the corresponding Partial differential equation. (3)
2. Find the Complete integral of the equation $zpq - p - q = 0$ (3)
3. Reduce the equation $4u_{xx} - 4u_{xy} + 5u_{yy} = 0$ to Canonical form. (3)
4. Determine the Regions where the equation $u_{xx} - 2x^2 u_{xz} + u_{yy} + u_{zz} = 0$ is of hyperbolic, elliptic or parabolic type. (3)
5. Show that the integral equation corresponding to boundary value problem, $\frac{d^2y}{dx^2} + \lambda y = 0,\ y(0) = 0,\ y(l) = 0$ is a Fredholm equation of the second kind. (3)
6. Show that characteristic numbers of a Fredholm equation with real symmetric kernel are all real. (3)

(PART B)

Answer Four questions from this part, without avoiding any unit.
Maximum Marks from this part is 48.

(UNIT I)

7. (a) Let $z = F(x, y, a)$ be a one parameter family of solutions of $f(x, y, z, p, q) = 0$. Show that this one parameter family, if it exists, is also a solution. (4)

   (b) Show that $(x-a)^2 + (y-b)^2 + z^2 = 1$ is a complete integral of $z^2(1 + p^2 + q^2) = 1$ (4)

   (c) Solve $x^2p + y^2q = (x + y)z$ (4)

8. (a) Obtain the necessary and sufficient condition of integrability of the Pfaffian differential equation $\vec{X}.\vec{dr} = P(x, y, z)dx + Q(x, y, z)dy + R(x, y, z)dz = 0$ (6)

   (b) Show that the necessary and sufficient condition for the integrability of $dz = \varphi(x, y, z)dx + \psi(x, y, z)dy$ is $[f, g] = 0$ (6)

9. (a) Find the Complete integral of $p^2x + q^2y = z$ (4)

   (b) Solve $u_x x^2 - u_y^2 - au_z^2 = 0$ using Jacobi's Method (3)

   (c) Solve $z = \frac{1}{2}(p^2 + q^2) + (p - x)(q - y)$ such that the integral passes through the x-axis (5)

(UNIT II)

10. (a) Derive d'Alembert's Solution of Wave Equation. (6)

(b) Derive the Riemann function and hence obtain the solution of $u_{\xi\eta} = 0$ (6)

11. (a) State and Prove maximum Principle. (4)

(b) Show that The solution of Neumann problem is unique up to the addition of a constant. (4)

(c) Discuss the Dirichlet problem for the Upper Half Plane (4)

12. (a) Solve the non - homogeneous wave equation $u_{tt} - c^2 u_{xx} = F(x, t), -\infty < x < \infty$ with homogeneous initial conditions $u(x, 0) = u_t(x, 0) = 0$ using Duhamel's Principle. (6)

(b) State and Prove Kelvin's inversion theorem (6) (

UNIT III)

13. (a) Solve $y'' + xy = 1, \; y(0) = 0, \; y(l) = 1$ using Green's function. (6)

(b) If $y_m(x)$ and $y_n(x)$ are characteristic functions of $y(x) = \lambda \int_a^b K(x, \xi) y(\xi)\, d\xi$ corresponding to distinct characteristic numbers , then show that $y_m(x)$ and $y_n(x)$ are orthogonal over the interval (a, b) (6)

14. (a) Show that the non-homogenous Fredholm integral equation of second kind with separable kernel can be reduced to a system of linear algebraic equations and hence show that such equation may have a unique solution or an infinite number of solutions. (6)

(b) Solve the Fredholm integral Equation $y(x) = F(x) + \lambda \int_0^1 (1 - 3x\xi) y(\xi)\, d\xi$ (6)

15. (a) Describe the iterative method for Solving Fredholm equation of the Second Kind (5)

(b) Solve by iterative method $y(x) = 1 + \lambda \int_0^1 (1 - 3x\xi) y(\xi)\, d\xi$ (3)

(c) Obtain the resolvent kernel associated with $K(x, \xi) = x\xi$ in (0, 1)

Model question paper

Third Semester MSc Degree Examination

MATHEMATICS

MAT 3 C 11: NUMBER THEORY

Time : 3 Hours Maximum Marks : 60

PART A

*Answer **any four** questions from Part **A**. Each question carries 3 marks*

1. Prove that $n^4 + 4$ is composite if $n > 1$.
2. Prove that $\phi(n) > \frac{n}{6}$ for all n with at most 8 distinct prime factors
3. Solve the congruence $5x \equiv 3 \pmod{4}$.
4. Determine those odd primes p for which $(-3 | p) = 1$ and for which $(-3 | p) = -1$.
5. Prove that 3 is a primitive root mod p if p is a prime of the form $2^n + 1$, $n > 1$.
6. Show that an algebraic integer is a rational number if and only if it is a rational integer.

PART B

*Answer **any four** questions from Part **B** not omitting any unit**.** Each question carries **12** marks.*

UNIT 1

7. (a) State and prove the fundamental theorem of arithmetic.

   (b) Show that the infinite series $\sum_{n=1}^{\infty} \frac{1}{p_n}$ diverges.

8. (a) If $n \geq 1$ show that $\phi(n) = \sum_{d|n} \mu(d) \frac{n}{d}$.

   (b) If f is an arithmetical function with $f(1) \neq 0$ show that there exists a unique arithmetical function f^{-1} such that $f * f^{-1} = f^{-1} * f = I$. Obtain also a recursive formula for f^{-1}.

9. (a) State and prove the Chinese remainder theorem.

   (b) Show that the set of lattice points in the plane visible from the origin contains arbitrarily large gaps.

UNIT 2

10. (a) State and prove Gauss' lemma.

(b) Determine whether 888 is a quadratic residue or nonresidue of the prime 1999.

11. (a) If m is a positive integer which is not of the form $m = 1, 2, 4, p^\alpha$, or $2p^\alpha$, where p is an odd prime, then show that for any a with $(a, m) = 1$ we have $a^{\phi(m)/2} \equiv 1 \pmod{m}$.

(b) Prove that 3 is a primitive root mod p if p is a prime of the form $2^n + 1,\ n > 1$.

12. (a) Explain the RSA public key algorithm with an example.

(b) Solve the superincreasing knapsack problem $28 = 3x_1 + 5x_2 + 11x_3 + 20x_4 + 41x_5$.

UNIT 3

13. (a) Let R be a ring. Show that every symmetric polynomial in $R[t_1, \cdots, t_n]$ is expressible as a polynomial with coefficients in R in the elementary symmetric polynomials $s_1, \cdots, s_n$.

(b) If K is a number field then $K = \mathbb{Q}(\theta)$ for some algebraic number θ.

14. (a) Show that every number field K possesses an integral basis and the additive group D is free abelian of rank n equal to degree of K.

(b) Let $K = \mathbb{Q}(\theta)$ be a number field where θ has minimum polynomial p of degree n. Show that the $\mathbb{Q}$-basis $\{1, \cdots, \theta^{n-1}\}$ has discriminant $\Delta[1, \theta, \cdots, \theta^{n-1}] = (-1)^{n(n-1)/2} N(D_p(\theta))$

15. (a) Let d be a square free rational integer. Then show that the integers of $\mathbb{Q}(\sqrt{d})$ are $\mathbb{Z}(\sqrt{d})$ if d is not congruent to 1 modulo 4 and $\mathbb{Z}\left(\frac{1}{2} + \frac{1}{2}\sqrt{d}\right)$ if $d \equiv 1 \pmod{4}$.

(b) Show that the ring of integers $Q(\varsigma)$ is $\mathbb{Z}(\varsigma)$