KANNUR UNIVERSITY

(Abstract)

B.Sc Mathematics Programme - Revised syllabus and model question paper under Choice Based Credit Semester System - Implemented w.e.f 2017 admissions **-** Orders Issued.

ACADEMIC C SECTION

No. Acad/C2/ 4762 /2014 **Dated, Civil Station P.O, 23-05-2017**

Read: 1. U.O of Even No. dated 12.05.2014

2. Minutes of the meeting of the BOS in Mathematics (UG) held on 20.12.2016.

3. Email from the Chairman , BOS in Mathematics (UG) dated 22.05.2017

ORDER

1. As per paper read (1) above, the scheme syllabus and pattern of question papers for core, complementary and open courses in B.Sc. Mathematics programme were implemented in the university w.e.f 2014 admission.

2. The meeting of the BOS in Mathematics (UG) held on 20.12.2016 vide paper read (2) above has recommended to incorporate certain modifications in the core papers 1B01MAT, 2B02MAT, 3B03MAT, 4B04MAT, 5B09MAT of B.Sc. Mathematics programme to be implemented w.e.f 2017 admissions.

3. The Chairman, Board of Studies in Mathematics (UG) vide paper read (3) above has submitted the revised syllabus of the core papers 1B01MAT, 2B02MAT, 3B03MAT, 4B04MAT, 5B09MAT of B.Sc. Mathematics programme to be implemented w.e.f 2017 admissions.

4. The Vice Chancellor, after examining the matter in detail, and in exercise of the powers of the Academic Council as per section 11(1) of Kannur University Act 1996 and all other enabling provisions read together with, has accorded sanction to implement with effect from 2017 admission, the revised syllabus of B.Sc. Mathematics programme incorporating the changes as recommended by the Board of Studies in Mathematics(UG), subject to report to the Academic Council.

P.T.O

5. The modified pages of syllabus and model question papers are appended for reference.

6. U.O as per the paper read (1) above, stands modified to this extent.

7. Orders, are therefore issued accordingly.

Sd/-
JOINT REGISTRAR (ACADEMIC)
FOR REGISTRAR

To

1. The Principals of the Affiliated Colleges offering B.Sc Mathematics course.

Copy To:

1. The Chairman, BOS in Mathematics (UG)
2. PS to VC/PA to PVC/PA to Registrar/PA to CE
3. JR/AR I Academic
4. SF/DF/FC.

Forwarded/By Order

Section Officer

For more details; log on www.kannuruniversity.ac.in

Modified Syllabus for B.Sc. Mathematics (Core)

1B01 MAT: Differential Calculus

Module-I (18 Hours)

Limit and continuity, The Sandwich theorem, Continuity. (Section 1.2and 1.5 of Text 3) Inverse functions and their derivatives, Hyperbolic functions and their derivatives (Section 6.1 and 6.10 of Text 3). Successive differentiation, Standard results - n^{th} derivatives, Leibnitz's theorem. (Sections 4.1 to 4.3 of Text 2)

Module II (18 Hours)

Polar co-ordinates, Cylindrical and spherical co-ordinates (Sections 9.6 and 10.7 of Text 3) Derivative of arc, curvature, radius of curvature (except the topic: radius of curvature for pedal curve), Centre of curvature, Evolute and Involute, (Sections 4.12 to 4.15 of Text 2).

Module-III (18 Hours)

Rolle's theorem, Lagrange's mean value theorem, Taylor's theorem, Maclaurin series, Taylor series, Increasing and decreasing functions, Maxima and minima, Asymptote (Sections 4.3 to 4.7, 4.17, 4.18, 4.20 of Text 2). Graphing with y' and y'' - concavity, point of inflection and graphing, L Hospital's rule - Indeterminate forms, (Section 3.4 and 6.6 of Text 3)

Module-IV (18 Hours)

Functions of several variables, Limits and continuity, Partial derivatives, Chain rule (theorem 5 without proof) (Sections 12.1 to 12.3, 12.5 of Text 3). Homogeneous functions, Euler's theorem on homogeneous functions. (Sections 11.8 and 11.8.1 of Text 1)

Texts:
1. S. Narayan and P. K. Mittal, Differential Calculus, Revised Edition, S. Chand Publishing.
2. B.S. Grewal, Higher Engineering Mathematics, 36th Edition
3. G. B. Thomas and R. L. Finney, Calculus and Analytic Geometry, 9th Edition.

References:
1. M. D. Weir, J. Hass and F. G. Giordano, Thomas' Calculus, 11th Edition, Pearson.
2. H. Anton, I. Bivens and S. Davis, Calculus, 7th Edition, Wiley.
3. S. K. Stein, Calculus with Analytic Geometry, McGraw Hill.
4. G. F Simmons, Calculus with Analytic Geometry, 2nd Edition, McGraw Hill.
5. S. S. Sastry, Engineering Mathematics, Vol. 1, 4th Edition, PHI.

Module	Teaching Hours	External Examination		Internal Mark	Total Mark	Credit
		Aggregate Mark	Maximum Mark			
I	18	18	12	12	60	4
II	18	18	12			
III	18	18	12			
IV	18	18	12			
Total	72	72	48	12	60	

2B02 MAT: Integral Calculus

Module – I (18 hours)

Riemann sum and definite integrals, Properties, Mean Value theorem for definite integrals, Fundamental theorem of calculus (without proof), Substitution in definite integrals, Integration of hyperbolic functions, Reduction formulae.
(Sections 4.5 to 4.8 , 6.10 and 7.5 of Text 1)

Module - II (15 hours)

Beta and gamma functions, introduction, The gamma function, Transformation of gamma function, Beta function, Evaluation of beta function, B(m,n)=B(n,m), Transformation of beta function, Relationship between beta and gamma functions, Gamma function and trigonometric relations. (Sections 10.1 to 10.9 of Text 2)

Module – III (21 hours)

Application of integration- Area between curves, Volume of solid of revolution length of curves, Length of parameterized curves, Area of surface of revolution, integration in parametric and polar co-ordinates. (Sections 5.1, 5.3, 5.5, 5.6, 9.5, 9.9 of Text 1)

Module - IV (18 hours)

Multiple integrals, Double integrals, area of bounded region in the plane, (except the topics: Moments and Centers of Mass, Centroids of Geometric Figures) double integral in polar form, triple integral in rectangular co ordinates, triple integral in cylindrical and spherical co-ordinates. (sections 13.1 to 13.4, 13.6 of Text 1)

Texts:
1. G. B. Thomas and R. L. Finney, Calculus, 9th Edition .
2. S. K. Sengar and S.P. Singh, Advanced Calculus, Cengage Learning India Pvt.

References:

1. S. Narayanan and T.K.M. Pillay, Differential and Integral Calculus, S. Viswanathan Printers and Publishers, Chennai.
2. H. Anton, I. Bivens and S. Davis, Calculus, 7th Edition, Wiley.
3. S. K. Stein, Calculus with Analytic Geometry, McGraw Hill.
4. M.R. Spiegel, Theory and Problems of Advanced Calculus, Schaum's Series.
5. S. S. Sastry, Engineering Mathematics, Vol. 1, 4th Edition, PHI

Module	Teaching Hours	External Examination		Internal Mark	Total Mark	Credit
		Aggregate Mark	Maximum Mark			
I	18	18	12	12	60	4
II	15	15	10			
III	21	21	14			
IV	18	18	12			
Total	72	72	48	12	60	

3B03 MAT: Elements of Mathematics I

Module – I (20 hours)

Finite and Infinite sets, Countable and uncountable sets, Cantor's theorem, Logic and proofs (Section 1.3 and Appendix A of text 3)

Arguments, Logical implications, Propositional functions, Quantifiers, Negation of quantified statements. (Sections 10.9 to 10.12 of Text 1)

Module – II (27 hours)

Basic concepts, Fundamental theorem of algebra (without proof), Relation between roots and coefficients, Symmetric functions of roots, Sum of the powers of roots, Newton's theorem on sum of the powers of roots, Transformation of equations, Reciprocal equations, Transformation in general. (Chapters 6: Sec 1 to 16 and 21 of Text 2)

Module - III (18 hours)

Descartes rule of signs, Multiple roots, Sturm's theorem, Cardon's method, Solution of biquadratic equation (Chapters 6: Sec 24, 26, 27, 34.1 and 35 of Text 2).

Module - IV (25 hours)

Divisibility theory in the integers – the division algorithm, the greatest common divisor, the Euclidean algorithm, the Diophantine equation ax + by = c. Primes and their distribution- fundamental theorem of arithmetic, the sieve of Eratosthenes. The theory of congruence- basic properties of congruence. (Sections 2.2, 2.3, 2.4, 2.5, 3.1, 3.2, 4.2of Text 4)

Texts:

1. S. Lipschitz, Set Theory and Related Topics, 2nd Edition, Schaum's series.
2. T. K. Manicavachagom Pillai, T. Natarajan and K. S. Ganapathy, Algebra Vol-1, S Viswanathan Printers and Publishers, 2010.
3. R. G. Bartle & Donald R. Sherbert, Introduction to Real Analysis, 3rd Edition, Wiley.
4. D. M. Burton, Elementary Number Theory, 7th Edition, TMH

References:

1. C.Y. Hsiung, Elementary Theory of Numbers, Allied Publishers.
2. N. Robbins, Beginning Number Theory, Second Edition. Narosa.
3. G. E. Andrews, Number Theory, HPC.
4. M.D. Raisinghnia and R.S. Aggarwal, Algebra.
5. K.H. Rosen, Discrete Mathematics and its Applications, 6th Edition, Tata McGraw Hill Publishing Company, New Delhi.

Module	Teaching Hours	External Examination		Internal Mark	Total Mark	Credit
		Aggregate Mark	Maximum Mark			
I	20	15	10	12	60	4
II	27	21	14			
III	18	15	10			
IV	25	21	14			
Total	90	72	48	12	60	

4B04 MAT: Elements of Mathematics II

Module I (25 Hours)

Relations, Types of relations, Partitions, Equivalence relation, Partial ordering relation, Functions, Composition of functions, One to one, Onto and invertible functions, Mathematical functions (except exponential and logarithmic functions), Recursively defined functions. (Sections 3.3, 3.6, 3.8, 3.9, 3.10 and chapter 4 of Text 1)

Module II (20 Hours)

Ordered sets, Partially ordered sets and Hasse diagrams, Minimal and maximal elements, First and last elements, Supremum and infimum, Lattices. Bounded, distributive, Complemented lattices. (Chapter 7: Sections 7.2, 7.4, 7.5, 7.7, 7.10, 7.11 of Text 1)

Module III (25 Hours)

Definition of conic, Parabola-ellipse-hyperbola, Some important results associated with the standard form of parabola-ellipse-hyperbola, General equation of parabola-ellipse-hyperbola, Position of a point with respect to a parabola-ellipse-hyperbola, Equation of tangent and normal at a point on a parabola-ellipse-hyperbola, Equation of chord of contact of a point with respect to a parabola-ellipse-hyperbola, Equation of pair of tangents from a given point, Parametric equation of a parabola-ellipse-hyperbola, Auxiliary circle and Eccentric angle, Rectangular hyperbola, Chord, tangent and normal of rectangular hyperbola, conjugate hyperbola. (Examples and problems after conormal points are not included --Sections 4.1 to 4.9, 5.1 to 5.9 and 6.1 to 6.12 of Text 2)

Module –IV (20 Hours)

Rank of a matrix – Elementary transformation, reduction to normal form, row reduced echelon form, computing the inverse of a non singular matrix using elementary row transformation. (Section 4.1 to 4.13 of Text 3)

Texts:
1. S. Lipschitz, Set Theory and Related Topics, 2nd Edition, Schaum's Series.
2. A. N. Das, Analytical Geometry of Two and Three Dimensions, NCBA, Reprint 2016.
3 S. Narayanan and Mittal, A Text Book of Matrices, Revised Edition, S. Chand.

References:
1. P. R. Vital, Analytical Geometry, Trigonometry and Matrices, Pearson Education
2. P.R. Halmos, Naive Set Theory, Springer.
3. E. Kamke, Theory of Sets, Dover Publishers.
4. D. Serre, Matrices, Theory and Applications, Springer.

Module	Teaching Hours	External Examination		Internal Mark	Total Mark	Credit
		Aggregate Mark	Maximum Mark			
I	25	22	15	12	60	4
II	20	14	9			
III	25	22	15			
IV	20	14	9			
Total	90	72	48	12	60	

5B09 MAT: Graph Theory

Module I – Basic Results(18Hours)

Introduction, Basic Concepts,Subgraphs, Degrees of Vertices,Paths and Connectedness,Line Graphs (Whitney's theorem without proof),Operations on Graphs. (Sections 1.1 to 1.8 except 1.6)

Module II –Connectivity,Trees (24 Hours)

Introduction, Vertex Cuts and Edges Cuts, Connectivity and Edge Connectivity (Whitney's theorem without proof), Introduction, Definition, Characterization, and Simple Properties, Centers and Centroids, Counting the Number of Spanning Trees, (Sections 3.1 to 3.3 and 4.1 to 4.4)

Module III – Independent Sets, Eulerian and Hamiltonian Graphs (18 Hours)

Introduction,Vertex-Independent Sets and Vertex Coverings, Edge-Independent Sets, Introduction, Eulerian Graphs, Hamiltonian Graphs, Hamilton's "Around the World" Game.(Sections 5.1 to 5.3, and 6.1 to 6.3 and 6.3.1)

Module IV – Directed Graphs (12 Hours)

Introduction,Basic Concepts, Tournaments (Sections 2.1 to 2.3)

Text: R. Balakrishnan and K. Ranganathan, A Text Book of Graph Theory, 2nd Edition, Springer

References:

1. J.A. Bondy and U.S.R.Murty, Graph Theory with applications. Macmillan
2. F. Harary, Graph Theory, Narosa publishers
3. J. Clark and D. A. Holton, A First look at Graph Theory, Prentice Hall
4. K.R. Parthasarathy, Basic Graph Theory, Tata-McGraw Hill
5. J.A. Dossey, Discrete Mathematics, Pearson Education.

Module	Teaching Hours	External Examination		Internal Mark	Total Mark	Credit
		Aggregate Mark	Maximum Mark			
I	18	18	12	12	60	3
II	24	24	16			
III	18	18	12			
IV	12	12	8			
Total	72	72	48	12	60	

Sd/-

Prof. Tom Joseph

Chairman, BOS in Mathematics (UG).

KANNUR UNIVERSITY MODEL QUESTION PAPER

FIRST SEMESTER B.Sc. DEGREE EXAMINATION

Mathematics (Core)

1B01MAT-Differential Calculus

Time: Three Hours | Maximum Marks: 48

Section A

All the first 4 questions are compulsory. They carry 1 mark each.

1. Find $\lim_{x\to c}\frac{x^3+4x^2-3}{x^2+5}$.

2. Fill in the blanks: $\frac{d}{dx}(\coth x) =$ ………………..

3. Find the Cartesian form of the polar equation

   $r=\frac{8}{1-2\cos\theta}$.

4. Find the polar coordinates corresponding to the Cartesian coordinate $\left(-3,\sqrt{3}\right)$.

Section B

Answer any 8 questions from among the questions 5 to 14. These questions carry 2 marks each.

5. Find $\lim_{h\to 0}\frac{\sqrt{2+h}-\sqrt{2}}{h}$.

6. Find the inverse of $y=\frac{1}{2}x+1,$ expressed as a function of x.

7. If $y=\sqrt{\sin x+\sqrt{\sin x+\sqrt{\sin x+\cdots}}}$ prove that

   $$\left(2y-1\right)^2 y_2+2y_1\cos x+\left(2y-1\right)\sin x=0.$$

8. Find the Cartesian and spherical co-ordinates of the point whose cylindrical coordinates is $(1,\ \pi/2,\ 1)$.

9. Translate the Cartesian equation $x^2+y^2+z^2=4z$ into two other forms.

10. Verify Rolle's Theorem for the function f defined by

$$f(x) = (x-a)^m (x-b)^n,$$

where m and n being positive integers and $x \in [a, b]$.

11. Using Maclaurin's series expand e^{2x}.

12. Find points of inflection on the curve $y = 3x^4 - 4x^3 + 1$.

13. For the function $f(x, y) = y - x$,

    (a) find the function's domain,

    (b) find the function's range, and

    (c) describe the function's level curves.

14. Let $f(x, y) = x^2 - xy + \frac{1}{2}y^2 + 12$. Verify Euler's Mixed Derivative Theorem at the point $(3, 2)$.

Section C

Answer any 4 questions from among the questions 15 to 20. These questions carry 4 marks each.

15. Show that

$$f(x) = \frac{x^2 + x - 6}{x^2 - 4}$$

is not continuous at $x = 2$, but has a continuous extension to $x = 2$, and find that extension.

16. Find the local and absolute extreme values of

$$f(x) = x^{\frac{1}{3}}(x-4) = x^{\frac{4}{3}} - 4x^{\frac{1}{3}}.$$

17. Find the asymptotes of the curve

$$y = \frac{x+3}{x+2}.$$

18. Using l'Hôpital's Rule, evaluate $\lim_{x \to 2^+} \frac{x^2 + 3x - 10}{x^2 - 4x + 4}$.

19. Using Chain Rule, find $\frac{dw}{dt}$ if

$$w = xy + z, \quad x = \cos t, \quad y = \sin t, \quad z = t.$$

What is the derivative's value at $t = \frac{\pi}{2}$.

20. Verify that $\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x}$, where $f = x^y$.

Section D

Answer any* 2 *questions from among the questions* 21 *to* 24. *These questions carry* 6 *marks each.

21. If $y = (\sin^{-1} x)^2$, prove that

$$(1 - x^2) y_{n+2} - (2n+1) x y_{n+2} - n^2 y_n = 0.$$

22. Graph the function $x^2 + y^2 + (z-1)^2 = 1$.

23. Find the centre of curvature and the evolute of the four cusped hypocycloid $x^{2/3} + y^{2/3} = a^{2/3}$.

24. If $u = \tan^{-1}\left(\frac{x^3 + y^3}{x - y}\right)$, $x \neq y$ show that

(i) $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \sin 2u.$

(ii) $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = (1 - 4\sin^2 u) \sin 2u.$

KANNUR UNIVERSITY MODEL QUESTION PAPER

SECOND SEMESTER B.Sc. DEGREE EXAMINATION

Mathematics (Core)

2B02MAT-Integral Calculus

Time: Three Hours | Maximum Marks: 48

Section A

All the first 4 questions are compulsory. They carry 1 mark each.

1. Define Gamma Integral.
2. State the Fundamental Theorem of Calculus.
3. Evaluate $\int_0^1 \int_0^2 xy(x-y)\,dx\,dy$
4. Evaluate $\int_0^1 \int_0^2 \int_0^2 x^2 yz \; dz \; dy \; dx \cdot$

Section B

Answer any 8 questions from among the questions 5 to 14. These questions carry 2 marks each.

5. Express the limit of Riemann sums

$$\lim_{\|P\| \to 0} \sum_{k=1}^{n} (3c_k^2 - 2c_k + 5)\Delta x_k$$

as an integral if P denotes a partition of the interval $[-1, 3]$.

6. Find the average value of $f(x) = 4 - x^2$ on $[0, 3]$. Does f actually take on this value at some point in the given domain?
7. Find the area of the region between the x-axis and the graph of $f(x) = x^3 - x^2 - 2x, \; -1 \le x \le 2$.
8. Evaluate $\int_0^{\pi/2} 2\sinh(\sin t)\cos t\,dt$.

9. Find $\int \tan^5 x dx$.

10. Express $\int_0^2 x(8-x^3)^{1/3} dx$ in terms of a Beta function.

11. Find the area between $y = \sec^2 x$ and $y = \sin x$ from 0 to $\pi/4$.

12. Find the area of the surface generated by revolving the arc of the catenary $y = c\cosh\frac{x}{c}$ from $x = 0$ to $x = c$ about the x-axis.

13. Find the length of the astroid

$$x = \cos^3 t, \quad y = \sin^3 t, \quad 0 \le t \le 2\pi$$

14. Evaluate

$$\iint_R e^{x^2+y^2} dy\, dx,$$

where R is the semicircular region bounded by the x-axis and the curve $y = \sqrt{1-x^2}$

Section C

Answer any 4 questions from among the questions 15 to 20. These questions carry 4 marks each.

15. Show that if f is continuous on $[a, b]$, $a \neq b$, and if $\int_a^b f(x)dx = 0$, then $f(x) = 0$ at least once in $[a, b]$.

16. Show that $\Gamma(1/2) = \sqrt{\pi}$.

17. Find the volume of the solid generated by revolving the region bounded by $y = \sqrt{x}$ and the lines y=1, x=4 about the line y=1.

18. For the catenary $y = c\cosh\frac{x}{c}$, show that $y^2 = c^2 + s^2$, where s is the length of the arc measured from its vertex to the point (x, y).

19. Change the order of integration and hence evaluate the double integral $\int_0^1 \int_{e^x}^{e} \frac{dx\, dy}{\log y}$

20. Change the following Cartesian integral into an equivalent polar integral and solve it.

$$\iint_R e^{x^2+y^2}\,dydx,$$

where R is the semi circular region bounded by the $x-$axis and the curve $y=\sqrt{1-x^2}$.

Section D

Answer any* 2 *questions from among the questions* 21 *to* 24. *These questions carry* 6 *marks each.

21. Show that

$$\int x\sin^{-1}x\,dx=\left(\frac{x^2}{2}-\frac{1}{4}\right)\sin^{-1}x+\frac{1}{4}x\sqrt{1-x^2}+C,$$

where C is an arbitrary constant.

22. To prove that $\beta(m,n)=\dfrac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$.

23. Find the area of the surface generated by revolving the right-hand loop of the lemniscate $r^2=\cos 2\theta$ about the $y-$axis.

24. Find the volume of the upper region *D* cut from the solid sphere $\rho\le 1$ by the cone $\phi=\pi/3$.

KANNUR UNIVERSITY MODEL QUESTION PAPER

THIRD SEMESTER B.Sc. DEGREE EXAMINATION

Mathematics (Core)

3B03MAT-Elements of Mathematics - I

Time: Three Hours | Maximum Marks: 48

Section A

All the first 4 questions are compulsory. They carry 1 mark each.

1. Fill in the blanks: If A is a set with $m \in \square$ elements and $C \subseteq A$ is a set with 1 element, then $A \setminus C$ is a set with ………….. elements.
2. Give the remainder obtained when a polynomial $f(x)$ is divided by $x-a$.
3. State Sturm' Theorem.
4. State True/False: Square of any integer is either $3k$ or $3k+1$.

Section B

Answer any 8 questions from among the questions 5 to 14. These questions carry 2 marks each.

5. Prove that if *A* and *B* are denumerable, then $A \cup B$ is denumerable.
6. Show that $\sqrt{2}$ is irrational.
7. Form the polynomial equation of fourth degree with rational coefficients, one of whose roots is $\sqrt{2}+\sqrt{-3}$.
8. If α, β and γ are the roots of the equation $x^3 + ax^2 + bx + c = 0$, form the equation whose roots are $\alpha\beta, \beta\gamma$ and $\gamma\alpha$.
9. If $\alpha, \beta, \gamma, \delta$ are the roots of

$$x^4 + px^3 + qx^2 + rx + s = 0,$$

find the value of $\sum \alpha^2\beta$.

10. Discuss the nature of roots of the equation

$$x^9+5x^8-x^3+7x+2=0.$$

11. Find the multiple roots of the equation $x^4-9x^2+4x+12=0.$

12. Show that the expression $\frac{a(a^2+2)}{3}$ is an integer for all $a\geq 1.$

13. If a and b are given integers, not both zero, then prove that the set

$$T=\{ax+by \mid x, y \text{ are integers}\}$$

is precisely the set of all multiples of $d=\gcd(a,b).$

14. Let $n>1$ be fixed and a, b be arbitrary integers. Then prove the following properties:

(a) $a\equiv a (\bmod\ n).$

(b) If $a\equiv b (\bmod\ n)$, then $b\equiv a (\bmod\ n).$

Section C

Answer any* 4 *questions from among the questions* 15 *to* 20. *These questions carry* 4 *marks each.

15. State and prove Cantor's Theorem.

16. Prove that in a polynomial equation with real coefficients imaginary roots occur in conjugate pairs.

17. Solve the reciprocal equation

$$60x^4-736x^3+1433x^2-736x+60=0.$$

18. Solve the equation $x^3+x^2-16x+20=0,$ given that some of its roots are repeated.

19. Prove that the linear Diophantine equation $ax+by=c$ has a solution if and only if $d \mid c,$ where $d=\gcd(a, b).$

20. Prove that there are infinitely many primes.

Section D

Answer any* 2 *questions from among the questions* 21 *to* 24. *These questions carry* 6 *marks each.

21. (a) Show that the propositions $p\to q$ and $\neg p\vee q$ are logically equivalent

(b) Use quantifiers to express the statement "There is a woman who has taken a flight on every airline in the world."

22. If α, β, γ are the roots of $x^3 - x - 1 = 0$, find the equation whose roots are

$$\frac{1+\alpha}{1-\alpha},\ \frac{1+\beta}{1-\beta},\ \text{and}\ \frac{1+\gamma}{1-\gamma}.$$

Hence write down the value of $\sum \frac{1+\alpha}{1-\alpha}$.

23. Solve the cubic

$$x^3 - 9x + 28 = 0$$

by Cardan's method.

24. State and prove the Fundamental Theorem of Arithmetic.

KANNUR UNIVERSITY MODEL QUESTION PAPER

FOURTH SEMESTER B.Sc. DEGREE EXAMINATION

Mathematics (Core)

4B04 MAT-Elements of Mathematics - II

Time: Three Hours | Maximum Marks: 48

Section A

All the first 4 questions are compulsory. They carry 1 mark each.

1. Let $A = \{1, 2\}$ and $B = \{a, b, c\}$. Then $A \times B =$ ……………….

2. Give the partition of the set $S = \{a,b,c,d\}$ that contain 4 distinct cells.

3. Give the rank of the matrix $\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$

4. Find the matrix that is obtained by multiplying second row of the following matrix by 7.

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 5 & 3 & 7 \\ 0 & 4 & 9 & 10 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Section B

Answer any 8 questions from among the questions 5 to 14. These questions carry 2 marks each.

5. Let *A* be a set of nonzero integers and let $\approx$ be the relation on $A \times A$ defined as follows:

$(a,b) \approx (c,d)$ whenever $ad = bc$

Prove that $\approx$ is an equivalence relation.

6. Let the function f and g be defined by $f(x) = 2x + 1$ and $g(x) = x^2 - 2$. Find the formula defining the composition functions: (a) $g \circ f$; (b) $f \circ g$

7. Let *n* denote a positive integer. Suppose a function *L* is defined recursively as follows:

$$L(n) = \begin{cases} 0 & \text{if } n = 1 \\ L(\lfloor n/2 \rfloor) + 1 & \text{if } n > 1 \end{cases}$$ where $\lfloor x \rfloor$ denotes the floor of x. Find L (25).

8. Define the following . Give one example to each:

    i. Bounded Lattice.

    ii. Distributive Lattice.

    iii. Non-distributive Lattice.

    iv. Join Irreducible elements.

9. Give an example of a collection S of sets ordered by set inclusion, and a subcollection $A=\{A_i : i \in I\}$ of S such that $B=\bigcup_i A_i$ is not an upper bound of A.

10. Find the focus, directrix, latus rectum and vertex of the parabola $y^2 = 4x$..

11. Find the equation of an ellipse whose focus is $(3,1)$ directrix is $x - y + 6 = 0$ and eccentricity is $\frac{1}{2}$.

12. Let $P(ct_1, c/t_1)$ and $Q(ct_2, c/t_2)$ be any two points on the rectangular hyperbola $xy = c^2$. Find the equation of the chord PQ.

13. Find the equations of conjugate hyperbola and auxiliary circle of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$.

14. Reduce to normal form the matrix $A = \begin{bmatrix} 2 & -2 & 0 & 6 \\ 4 & 2 & 0 & 2 \\ 1 & -1 & 0 & 3 \\ 1 & -2 & 1 & 2 \end{bmatrix}$

Section C

Answer any 4 questions from among the questions 15 to 20. These questions carry 4 marks each.

15. Suppose $P = \{A_i\}$ is a partition of a set S. Then there is an equivalence relation R on S such that the quotient set S/R of equivalence classes is same as the partition $P = \{A_i\}$..

16. Determine if each of the following functions is one-to-one:

    (a) To each person on the earth assign the number which corresponds to his age.

    (b) To each country in the world assign the latitude and longitude of its capital.

    (c) To each state in India assign the name of its capital.

    (d) To each book written by only one author assign the author.

(e) To each country in the world which has a prime minister assign its prime minister.

17. Let L be a lattice. Prove the following:

i. $a \wedge b = a \Leftrightarrow a \vee b = b.$

ii. The relation $a \leq b$ (defined by $a \wedge b = a$) is a partial order on L.

18. Find the equation of chord of contact of tangents from (h, k) to the parabola $y^2 = 4ax$..

19. Obtain the equation of the pair of tangents from (h, k) to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

20. Find the rank of the following matrix by reducing it to the row reduced echelon form:

$$A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix}$$

Section D

Answer any* 2 *questions from among the questions* 21 *to* 24. *These questions carry* 6 *marks each.

21. Consider the set $\mathbb{Z}$ of integers. Define aRb if $b = a^r$ for some positive integer r. Show that R is a partial ordering on $\mathbb{Z}$.

22. Let L be a finite distributive lattice. Then show that every a in L can be written uniquely as the join of redundant join-irreducible elements.

23. Find the equation of the tangent and normal at a point (h, k) on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$.

24. Using elementary row transformations, compute the inverse of the matrix

$$A = \begin{bmatrix} -1 & -3 & 3 & -1 \\ 1 & 1 & -1 & 0 \\ 2 & -5 & 2 & -3 \\ -1 & 1 & 0 & 1 \end{bmatrix}.$$

KANNUR UNIVERSITY MODEL QUESTION PAPER

FOURTH SEMESTER B.Sc. DEGREE EXAMINATION

Mathematics (Core)

5B09 MAT-Graph Theory

Time: Three Hours | Maximum Marks: 48

Section - A

(Answer all the Questions, Each Question carries one marks.)

1. State the Whitney's theorem.
2. Define Euler Graphs.
3. Define Tournaments.
4. A simple graph G with n vertices, $n \geq 2$; is complete if and only if $\kappa(G) =$

Section - B

(Answer any Eight Questions, Each Question carries two Marks.)

5. Prove that the number of vertices of odd degree is even.
6. Prove that if a simple graph G is not connected then G^C is connected.
7. Prove that every connected graph contains a spanning tree.
8. Prove that an edge $\boldsymbol{e} = xy$ is a cut edge of a connected graph G if and only if there exist vertices u and v such that e belongs to every $u - v$ path in G.
9. Prove that for any graph G with $\boldsymbol{n}$ vertices, $\alpha + \beta = n$
10. Define Digraph, in degree and out degree with an example.
11. Prove that every tournament contains a directed Hamiltonian path.
12. Explain Directed Walk, Directed path, and Directed cycle.
13. Explain disconnected in a Digraph.
14. A subset S of V is independent if and only if $V - S$ is a covering of G.

Section - C

(Answer any four Questions, Each Question carries four Marks.)

15. Explain the different operations on Graphs with examples.

16. Prove that the line graph of a simple graph G is a path if and only if G is a path.

17. Prove that the number of edges in a tree on n vertices is $n - 1$.

18. Prove that for a connected a graph G with at least two vertices contains at least two vertices that are not cut vertices.

19. Prove that for a simple graph G with $n \geq 3$ vertices, if for every pair of nonadjacent vertices u, v of G, $d(u) + d(v) \geq n$, then G is Hamiltonian.

20. Prove that every vertex of a disconnected tournament T on n vertices with $n \geq 3$ is contained in a directed k -cycle, $3 \leq k \leq n$.

Section - D

(Answer any two Questions, Each Question carries six Marks.)

21. a) Define bipartite Graph.

    b) Prove that a Graph G is bipartite if and only if it contains no odd cycles.

22. Prove that for any loop less connected graph G, $\kappa(G) \leq \lambda(G) \leq \delta(G)$

    Give an example with strict inequality hold.

23. a) Define Centre and Centroids of a Graph.

    b) Prove that every tree has centre consisting of either a single vertex or two adjacent vertices

24. Prove that for any non trivial connected graph G, the following statements are equivalent:

    a) G is Eulerian.

    b) The degree of each vertex of G is an even positive integer.

    c) G is an edge disjoint union of cycles.

Tom Joseph