K23P 3114

Reg. No. : ....

Name : ....

I Semester M.Sc. Degree (CBCSS – OBE – Regular)
Examination, October 2023
(2023 Admission)
MATHEMATICS
MSMAT 01C04 : Topology

Time : 3 Hours Max. Marks : 80

PART – A

Answer **any five** questions. **Each** question carries **4** marks.

1. Give a basis for the standard topology on $\mathbb{R}^2$.
2. Let Y be a subspace of X and A be a subset of Y. Show that A is closed in Y if and only if it equals the intersection of a closed set of X with Y.
3. Show that limits of sequences are unique in a Hausdorff space.
4. Prove or disprove : Product topology is finer than the box topology.
5. Define a quotient map and give an example.
6. Is every path connected space connected ? Justify your answer. **(5×4=20)**

PART – B

Answer **any three** questions. **Each** question carries **7** marks.

7. Let $X = \{1, 2, 3, 4, 5\}$ and $\mathcal{T} = \{\phi, X, \{2, 5\}, \{2, 3, 4\}, \{2, 3, 4, 5\}, \{1, 2, 5\}, \{3, 4\}, \{2\}\}$
   a) List the closed subsets of X.
   b) Determine the closure of the sets {2,4} and {1, 3}.
   c) Check whether the space $(X, \mathcal{T})$ is T_2.
   d) Check whether the space $(X, \mathcal{T})$ is connected.

P.T.O.

8. Let

$d_1((x_1, x_2), (y_1, y_2)) = [(x_1 - y_1)^2 + (x_2 - y_2)^2]^{1/2}$, $d_2((x_1, x_2), (y_1, y_2)) = \max\{|x_1 - y_1|, |x_2 - y_2|\}$ and $d_3((x_1, x_2), (y_1, y_2)) = \min\{1, [(x_1 - y_1)^2 + (x_2 - y_2)^2]^{1/2}\}$ and $\mathcal{T}_1$, $\mathcal{T}_2$ and $\mathcal{T}_3$ be the corresponding topologies on $\mathbb{R}^2$ induced by the metrics d_1, d_2 and d_3 respectively. Which of the following options are correct ? Justify your answer.

a) $\mathcal{T}_1 = \mathcal{T}_2 \neq \mathcal{T}_3$

b) $\mathcal{T}_1 \neq \mathcal{T}_2 = \mathcal{T}_3$

c) $\mathcal{T}_1 \neq \mathcal{T}_2 \neq \mathcal{T}_3$

d) $\mathcal{T}_1 = \mathcal{T}_2 = \mathcal{T}_3$

9. Consider the subset $A = \{(x \times \sin\frac{1}{x}) : 0 < x \leq 1\}$ of the plane $\mathbb{R}^2$.

a) Find the closure of A in $\mathbb{R}^2$.

b) Determine the connectedness and path connectedness of closure of A.

10. Which of the following are topological property ?

a) connectedness

b) boundedness

c) pathconnectedness.

11. Determine whether the following subspaces of $\mathbb{R}$ are homeomorphic.

a) [0,1) and (0,1)

b) $\mathbb{Q}$ and $\mathbb{Z}$

c) (0,1) and $S^1 \setminus \{(1, 0)\}$ where $S^1 = \{x \times y : x^2 + y^2 = 1\}$ considered as a subspace of $\mathbb{R}^2$.

(3×7=21)

PART – C

Answer **any three** questions. **Each** question carries **13** marks.

12. Let X and Y be two topological spaces.

    a) If A is a subspace of X and B is a subspace of Y, prove that the product topology on $A \times B$ is the same as the topology $A \times B$ inherits as the subspace of $X \times Y$.

    b) If Π_1 and Π_2 are projections of $X \times Y$ to X and Y respectively, then prove that the collection $\{\Pi_1^{-1}(U)$: U is open in X$\}$ U $\{\Pi_2^{-1}(V)$: V is open in Y$\}$ is a sub-basis for the product topology on $X \times Y$.

13. a) Let X and Y be topological spaces and $f : X \to Y$. Prove that the following are equivalent.

    i) f is continuous.

    ii) For every subset A of X, $f(\bar{A}) \subset \overline{f(A)}$.

    iii) For every closed set B of Y, the set $f^{-1}(B)$ is closed in X.

    b) Let $X = A \cup B$ where A and B are closed in X and $f : A \to Y$ and $g : B \to Y$ be continuous. If $f(x) = g(x)$ for every $x \in A \cap B$, then prove that f and g combine to give a continuous function $h : X \to Y$ defined by $h(x) = f(x)$ if $x \in A$ and $h(x) = g(x)$ if $x \in B$.

14. a) Define order topology and linear continuum.

    b) Give an example of a linear continuum.

    c) Show that a linear continuum in the order topology is connected.

15. a) Show that $\mathbb{R}^\omega$ in product topology is metrizable.

    b) Is $\mathbb{R}^\omega$ connected in the product topology ? Give details to support your assertion.

16. a) Prove that the composition of two quotient maps is a quotient map.

    b) Let X and Y be topological spaces and $p : X \to Y$ be a quotient map. Let Z be a topological space and $g : X \to Z$ be a map that is constant on each set $p^{-1}(\{y\})$. Show that g induces a map $f : Y \to Z$ such that $f \circ p = g$ and f is a quotient map if and only if g is a quotient map. **(3×13=39)**

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