

Second Semester FYUGP Mathematics Examination
APRIL 2025 (2024 Admission onwards)
KU2DSCMAT116 (MULTIVARIABLE CALCULUS)
 (DATE OF EXAM : 02-05-2025)

Time : 120 min

Maximum Marks : 70

Part A (Answer any 6 questions. Each carries 3 marks)

1. Find the domain for the function $f(x, y) = \cos^{-1}(y - x^2)$. 3
2. Find $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ of $f(x, y) = e^{xy} \ln y$. 3
3. Find f_{yy} if $f(x, y) = \ln(x + y)$. 3
4. Find $\frac{d\mathbf{v}}{dt}$, for the vector $\mathbf{v}(t) = t^2\mathbf{i} + 2t\mathbf{j}$ for $t = 2$. 3
5. Define gradient of a scalar function $f(x, y, z)$. 3
6. Find the work done by the conservative field $\mathbf{F} = \nabla f$, where $f(x, y, z) = xyz$, along any smooth curve C joining $(-1, 3, 9)$ to $(1, 6, -4)$. 3
7. Show that $\mathbf{F} = (y \sin z)\mathbf{i} + (x \sin z)\mathbf{j} + (xy \cos z)\mathbf{k}$ is conservative. 3
8. What are the Moments of inertia of a thin shell of density $\delta(x, y, z)$ about coordinate axes. 3

Part B (Answer any 4 questions. Each carries 6 marks)

9. Find $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ if $f(x, y) = \left(x^3 + \frac{y}{2}\right)^{2/3}$. 6
10. Verify that $W_{xy} = W_{yx}$, where $W = \ln(2x + 3y)$. 6
11. Find f_{xx} , f_{yy} , f_{xy} and f_{yx} if $f(x, y) = x \cos y + ye^x$ 6
12. Evaluate $\int_C \sqrt{(x^2 + y^2)} ds$ along the curve $\mathbf{r}(t) = (4 \cos t)\mathbf{i} + (4 \sin t)\mathbf{j} + 3t\mathbf{k}$, $-2\pi \leq t \leq 2\pi$. 6
13. Find a parametrization of the cylinder $x^2 + (y - 3)^2 = 9$, $0 \leq z \leq 5$. 6
14. Integrate $G(x, y, z) = x^2$ over the cone $z = \sqrt{x^2 + y^2}$, $0 \leq z \leq 1$. 6

Part C (Answer any 2 question(s). Each carries 14 marks)

15. Find the volume of the solid whose base is the region in the xy-plane that is bounded by the parabola $y = 4 - x^2$ and the line $y = 3x$ while the top of the solid is bounded by the plane $z = x + 4$. 14

16. (a) Find the volume of the region bounded above by the plane $z = y/2$ and below by the rectangle $R : 0 \leq x \leq 4, 0 \leq y \leq 2$.
- (b) Evaluate $\int_0^1 \int_0^{y^2} 3y^3 e^{xy} dx dy$.

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17. (a) Find the unit normal vector to the surface $z^2 = x^2 + 2y^2$ at the point $(1, \sqrt{2}, \sqrt{5})$.
- (b) Find the directional derivative of the function $f(x, y, z) = x \sin(zy)$ at the point $\left(1, 1, \frac{\pi}{2}\right)$ in the direction of the vector $\mathbf{a} = [2, 3, -4]$.
- (c) Find the divergence of the vector function $[4z^2, -x^2, -y^2]$ at the point $(0, 1, -2)$.

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