

Second Semester FYUGP Mathematics Examination
APRIL 2025 (2024 Admission onwards)
KU2DSCMAT113 (SET THEORY, NUMBER THEORY,
INTEGRAL CALCULUS AND FOURIER SERIES)
 (DATE OF EXAM: 30-4-2025)

Time : 120 min

Maximum Marks : 70

Part A (Answer any 6 questions. Each carries 3 marks)

1. Define **empty set**. Give an example for empty set. 3
2. (a) Define symmetric difference of two sets A and B .
 (b) Draw the Venn diagram to represent the symmetric difference of A and B . 3
3. Define union and intersection of two sets. 3
4. Compute $\int_0^{\frac{\pi}{2}} \sin^3 x \, dx$, using reduction formula. 3
5. Evaluate $\int \cos^4 x \, dx$, using reduction formula. 3
6. What is the fundamental period of the function $f(x) = \cos 3x$? 3
7. If $f(x)$ is an even function, write its Fourier series of period 2π ? 3
8. If $f(x)$ is an even function, write its Fourier series of period $2L$? 3

Part B (Answer any 4 questions. Each carries 6 marks)

9. For the set $A = \{x \mid x \text{ is an even number less than } 10\}$, write the set explicitly and list all the subsets of A . 6
10. For the set $A = \{x \mid x \text{ is a positive integer less than or equal to } 10\}$ and the set $B = \{x \mid x \text{ is a multiple of } 3 \text{ less than or equal to } 10\}$, answer the following:
 (a) Write the sets A and B .
 (b) Find the power set of B .
 (c) Calculate $|A|$, $|B|$ and $|P(B)|$. 6
11. Let $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$, $A = \{1, 2, 4, 6, 8\}$, $B = \{2, 4, 5, 9\}$, $C = \{x \mid x \text{ is a positive integer and } x^2 \leq 16\}$, and $D = \{7, 8\}$. Compute
 (a) $A \cup B \cup C$
 (b) $A \cap (\overline{C} \cup D)$
 (c) $B \oplus C$. 6
12. Determine the Fourier series coefficient a_n in the Fourier series expansion $a_0 + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$ of the function $f(x) = \begin{cases} 1 & -\pi < x < 0 \\ -1 & 0 < x < \pi \end{cases}$ and $f(x + 2\pi) = f(x)$. 6

13. Determine the Fourier series coefficient b_n in the Fourier series expansion $a_0 + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$ of the function $f(x) = 1 + x$ on $[-\pi, \pi]$. 6

14. Obtain the Fourier coefficients a_0 and a_n in the Fourier series expansion $a_0 + \sum_{n=1}^{\infty} \left(a_n \cos \frac{n\pi}{L}x + b_n \sin \frac{n\pi}{L}x \right)$ of the 2-periodic function defined by:

$$f(x) = \begin{cases} 0, & \text{when } -1 < x < 0 \\ 1, & \text{when } 0 < x < 1. \end{cases}$$

6

Part C (Answer any 2 question(s). Each carries 14 marks)

15. Find $d = GCD(a, b)$ and write d as $sa + tb$ where $a = 108$, $b = 60$. 14
16. Find $d = GCD(a, b)$ and write d as $sa + tb$ where $a = 60$, $b = 100$. 14
17. Evaluate $\int \cos^{12} x \, dx$. 14