

Second Semester FYUGP Mathematics Examination**APRIL 2025 (2024 Admission onwards)****KU2DSCMAT101 (CALCULUS II)****(DATE OF EXAM: 28-4-2025)**

Time : 120 min

Maximum Marks : 70

Part A (Answer any 6 questions. Each carries 3 marks)

1. Define critical points of a function. 3
2. Using Mean Value theorem, prove the power rule for logarithm: $\ln x^r = r \ln x$. 3
3. Does the function $f(x) = |x|$ satisfy Rolle's theorem on $[-1, 1]$. Justify your answer. 3
4. Evaluate $\int_0^{\pi/2} \sin^{10} x \, dx$. 3
5. Evaluate $\int \cos^5 x \, dx$. 3
6. Evaluate $\int \tan^4 x \, dx$. 3
7. Find and sketch the level curve $f(x, y) = 1$, where $f(x, y) = x + y - 1$. 3
8. Find the domain for the function $f(x, y) = \frac{\sin(xy)}{x^2 + y^2 - 25}$. 3

Part B (Answer any 4 questions. Each carries 6 marks)

9. If $x = 2 \cos t - \cos 2t$ and $y = 2 \sin t - \sin 2t$, find the value of $\frac{d^2y}{dx^2}$ when $t = \frac{\pi}{2}$. 6
10. Determine the n^{th} derivative of $\sinh 2x \sin 4x$. 6
11. Obtain the Maclaurin's series expansion of $\sinh x$. 6
12. Obtain reduction formula for $\int \frac{x^n}{(\log x)^m} \, dx$. 6
13. Find the value of $\int_0^3 \sqrt{\frac{x^3}{3-x}} \, dx$. 6
14. Evaluate $\int_0^{\pi/4} \cos^3 2x \sin^4 4x \, dx$. 6

Part C (Answer any 2 question(s). Each carries 14 marks)

15. Compute the following limits.

$$(i) \lim_{\theta \rightarrow \frac{\pi}{2}} \frac{2\theta - \pi}{\cos(2\pi - \theta)} \quad (ii) \lim_{x \rightarrow 1} \frac{x - 1}{\ln x - \sin \pi x} \quad (iii) \lim_{x \rightarrow 0} \frac{x^2}{\ln(\sec x)}. \quad 14$$

16. You have been asked to design a one-liter can shaped like a right circular cylinder. What dimensions will use the least material? 14

17. (a) Use the chain rule to find the derivative of $w = xy$ with respect to t along the path $x = \cos t, y = \sin t$. Determine the value of the derivative at $t = \pi/2$.

(b) Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ at $(0, 0, 0)$ if $x^3 + z^2 + ye^{xz} + z \cos y = 0$.

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