First Semester FYUGP Mathematics Examination
November 2024 (2024 Admission onwards)
KU1DSCMAT116 (CALCULUS AND COORDINATE SYSTEMS)

(EXAM DATE : 06-12-2024)

Time : 120 min   Maximum Marks : 70

Part A (Answer any 6 questions. Each carries 3 marks)

1. Use the laws of exponents to simplify the following expressions:

   (a)$4^{\frac{1}{3}}.4^{\frac{1}{6}}$

   (b) $(8^{\frac{1}{9}})^3$.   3

2. Simplify the expression: $\ln(e^{2\ln x})$.   3

3. Find $\lim\limits_{y\to 0} \dfrac{y^2}{y^3+6}$.   3

4. Apply Chain rule to differentiate $y = e^{cosx}$.   3

5. If $g(t) = \dfrac{1}{t^2}$, find $g'(t)$ at $t = -1$.   3

6. State the Mean Value Theorem for definite integrals.   3

7. Evaluate $\displaystyle\int_0^{3b} x^2 \, dx$.   3

8. Evaluate $\displaystyle\int a \sin bx \, dx$.   3

Part B (Answer any 4 questions. Each carries 6 marks)

9. If $f(x) = \dfrac{x+2}{x-1}$, find $f^{-1}(x)$ and identify the domain and range of $f^{-1}(x)$ 6

10. Calculate the value of the limit $\lim\limits_{v\to 2} \dfrac{v^2-4}{v^4-16}$.   6

11. Find a closed-form for the inverse hyperbolic function

$$y = \tanh^{-1} x.$$

6

12. Evaluate $\displaystyle\int_0^{\frac{\pi}{6}} (\sec x + \tan x)^2 \, dx$.   6

13. Evaluate $\displaystyle\int \dfrac{1}{x(x+1)} dx$ 6

14. Evaluate $\dfrac{d}{dx} \displaystyle\int_0^3 (t^3 + 1)dt$ 6

Part C (Answer any 2 question(s). Each carries 14 marks)

15.   (a) Graph the curve $r = 1 + \cos\frac{\theta}{2}$

    (b) Describe the set of points $P\ (\rho, \phi,\ \theta)$ whose spherical co-ordinates satisfy the equations $\rho = 1$, $\phi = \frac{\pi}{3}$.

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16.   (a) Find the Cartesian equivalent of the polar equation $r = 1 + 2r\cos\theta$

    (b) Translate the equation $x^2 + y^2 + z^2 = 4z$ from the given coordinate system into equations in the other two coordinate systems.

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17.   (a) Find $\dfrac{dy}{dx}$ using the method of logarithmic differentiation, if $y = (x^2 + 1)(x^4 + 2)^{\frac{1}{2}}$.

    (b) Find $\dfrac{dy}{dx}$ using the method of logarithmic differentiation, if $y = \dfrac{x^5 + 5}{(x + 3)^2}$.

    (c) Show that there is a root of the equation $x^3 - x - 1 = 0$ between 1 and 2 .

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