Reg No:..... K24FY 1474 (B)
Name :.....

# First Semester FYUGP Mathematics Examination NOVEMBER 2024 (2024 Admission onwards) KU1DSCMAT113 (FUNCTIONS, CALCULUS AND MATRICES)

(DATE OF EXAM: 4-12-2024)

---

Time : 120 min Maximum Marks : 70

**Part A (Answer any 6 questions. Each carries 3 marks)**

1. Use the laws of exponents to simplify the following expressions:
   (a) $2^{\sqrt{3}}.7^{\sqrt{3}}$
   (b) $\left(\frac{2}{\sqrt{2}}\right)^2$. 3

2. Find the inverse of $y = \frac{x}{2} + 1$. 3

3. Evaluate $\lim_{x\to 0} \dfrac{1 + x + \sin x}{3\cos x}$. 3

4. Evaluate $\int a^{2x}\, dx$. 3

5. Evaluate $\int_0^{\frac{\pi}{4}} \sec^2 x dx$. 3

6. Find the transpose of
$$\begin{bmatrix} 1 & 5 & 6 & 3 \\ 2 & 5 & 7 & 8 \\ 5 & 9 & 2 & 4 \end{bmatrix}.$$
3

7. Define elementary matrices. 3

8. Find the rank of the matrix
$$A = \begin{bmatrix} 1 & 3 \\ 0 & -1 \end{bmatrix}.$$
3

**Part B (Answer any 4 questions. Each carries 6 marks)**

9. Express the following logarithms in terms of $\ln 5$ and $\ln 7$
   (a) $\ln(1/125)$ (b) $\ln(9.8)$ (c) $\ln(7\sqrt{7})$. 6

10. If $\sqrt{5-2x^2} \le f(x) \le \sqrt{5-x^2}$ for $-1 \le x \le 1$, find $\lim_{x\to 0} f(x)$, using the Sandwich Theorem. 6

11. If $2-x^2 \le g(x) \le 2\cos x$ for all $x$, find $\lim_{x\to 0} g(x)$, using the Sandwich Theorem. 6

12. Express the matrix $A$ as the sum of a symmetric and a skew-symmetric matrix where

$$A = \begin{bmatrix} 4 & 2 & -3 \\ 1 & 3 & -6 \\ -5 & 0 & -7 \end{bmatrix}.$$

6

13. Express the matrix $A$ as the sum of a symmetric and a skew-symmetric matrix where

$$A = \begin{bmatrix} a & a & b \\ c & b & b \\ c & a & c \end{bmatrix}.$$

6

14. Find the rank of the matrix

$$\begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}.$$

6

**Part C (Answer any 2 question(s). Each carries 14 marks)**

15. a) Suppose $u$ and $v$ are functions of $x$ that are differentiable at $x = 0$ and that $u(0) = 5, u'(0) = -3, v(0) = -1, v'(0) = 2$. Find the values of the following derivatives at $x = 0$.

(i) $\dfrac{d}{dx}(uv)$

(ii) $\dfrac{d}{dx}(7v - 2u)$.

b) Calculate the derivative $\dfrac{d}{dx}(\cos^{-1}(x^2))$. 14

16. (a) Find the first and second derivative of the function $y = \dfrac{(x^2 + x)(x^2 - x + 1)}{x^4}$.

(b) Evaluate $\dfrac{d}{dx}(\ln \sin x)$.

14

17. (a) Evaluate $\displaystyle\int \frac{(x+1)(x + \log x)^2}{2x} dx$.

(b) Evaluate $\displaystyle\int \sin^3 x \, dx$.

14