First Semester FYUGP Mathematics Examination
NOVEMBER 2024 (2024 Admission onwards)
KU1DSCMAT112 (CALCULUS AND MATRIX ALGEBRA)
(DATE OF EXAM: 4-12-2024)

Time : 120 min   Maximum Marks : 70

Part A (Answer any 6 questions. Each carries 3 marks)

1. Find $\dfrac{d}{dx}(e^x \sin x)$.   3

2. Find the derivative of $\ln 3x$ w.r.t. x.   3

3. Find $\dfrac{d}{dx}\left(3^{-x}\right)$.   3

4. Determine $\displaystyle\int \dfrac{1}{2^x}\,dx$.   3

5. Determine the integral $\displaystyle\int x^2 e^{x^3}\,dx$.   3

6. Reduce the matrix $\begin{bmatrix} 5 & \sqrt{3} \\ 0 & 0 \end{bmatrix}$ to its normal form, and hence find the rank.   3

7. Find the inverse of the 2×2 matrix $\begin{bmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{bmatrix}$.   3

8. Find the characteristic equation of the matrix $A = \begin{bmatrix} -2 & 1 \\ 1 & 2 \end{bmatrix}$.   3

Part B (Answer any 4 questions. Each carries 6 marks)

9. Find an equation for a line that is tangent to the graph of $y = e^x$ and goes through the origin.   6

10. Evaluate $\displaystyle\lim_{x \to \pi/2} \cos\left(2x + \sin\left(\dfrac{3\pi}{2} + x\right)\right)$.   6

11. Use implicit differentiation to find $\dfrac{d^2y}{dx^2}$ if $5x^3 - 7y^2 = 10$.   6

12. Find the inverse of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 0 & -1 & 2 \\ 2 & 4 & -1 \end{bmatrix}$ 6

13. Find the eigen vectors of the matrix $\begin{bmatrix} a & b \\ 0 & c \end{bmatrix}$.   6

14. Find the eigen vectors of the matrix $\begin{bmatrix} 4 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix}$.   6

Part C (Answer any 2 question(s). Each carries 14 marks)

15. (a)Find the inverse of the function $y = x^2 + 1$, $x \geq 0$, expressed as a function of x.
    (b)State Algebraic properties of the Natural Logarithm.
    (c)Simplify $3 \ln \sqrt[3]{t^2 - 1} - \ln(t + 1)$.   14

16. (a)Find the domain and range for the function $g(t) = \cos(e^{-t})$.
    (b)Prove that $\cosh^2 x - \sinh^2 x = 1$.
    (c)Compute $\lim\limits_{x \to 1} \dfrac{x^2 + x - 2}{x^2 - x}$.   14

17. (a) Evaluate $\displaystyle\int_0^1 \dfrac{x^5}{1 + x^{12}}\, dx$.

    (b) Evaluate $\displaystyle\int \cos^3 x \, dx$.

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