K25P 2893

Reg. No. : ....

Name : ....

III Semester M.Sc. Degree (C.B.C.S.S. – O.B.E. – Reg./Supple./Imp.) Examination, October 2025
(2023 Admission Onwards)
MATHEMATICS/MATHEMATICS (MULTIVARIATE CALCULUS AND MATHEMATICAL ANALYSIS, MODELLING AND SIMULATION, FINANCIAL RISK MANAGEMENT)
MSMAT03C11/MSMAF03C11 : Functional Analysis

Time : 3 Hours Maximum Marks : 80

PART – A

Answer **any five** questions. **Each** question carries **4** marks. **(5×4=20)**

1. Show that equivalent norms on a vector space X induce the same topology for X.

2. Let X and Y be metric spaces and $T : X \to Y$ a continuous mapping. Prove that the image of a compact subset M of X under T is compact.

3. Determine the null space of the operator $T : \mathbb{R}^3 \to \mathbb{R}^2$ represented by $\begin{bmatrix} 1 & 3 & 2 \\ -2 & 1 & 0 \end{bmatrix}$.

4. Let T be a linear operator. Prove that the range $\mathscr{R}(T)$ and the null space $\mathscr{N}(T)$ are vector spaces.

5. Prove that an orthonormal set is linearly independent.

6. Prove that every vector space $X \neq \{0\}$ has a Hamel basis.

PART – B

Answer **any three** questions. **Each** question carries **7** marks. **(3×7=21)**

7. State and prove Riesz Lemma.

8. Define equivalent norms. In $\mathbb{R}^n$, define $\|x\|_1 = |\xi_1| + |\xi_2| + \ldots + |\xi_n|$ and $\|x\|_2 = \left(|\xi_1|^2 + |\xi_2|^2 + \ldots + |\xi_n|^2\right)^{\frac{1}{2}}$, where $x = (\xi_1, \ldots, \xi_n) \in \mathbb{R}^n$. Prove that $\|x\|_1$ and $\|x\|_2$ are equivalent.

P.T.O.

9. Let $T : C[0,1] \to : C[0,1]$ be defined by $y = Tx$, where $y(t) = \int_0^1 k(t, \tau)\, x(\tau)\, d\tau$. Prove that T is a bounded linear operator.

10. State and prove Schwarz inequality.

11. Define self adjoint operator. Prove that the set of all self adjoint operators on a Hilbert space is a closed subspace.

PART – C

Answer **any three** questions. **Each** question carries **13** marks. **(3×13=39)**

12. Prove that $\ell^p = \left\{ x = (\xi_1, \xi_2, ...) \sum_{j=1}^{\infty} |\xi_j|^p < \infty \right\}$ is a Banach space with norm given by $\|x\| = \left(\sum_{j=1}^{\infty} |\xi_j|^p \right)^{\frac{1}{p}}$.

13. (a) Prove that B(X, Y) of all bounded linear operators from a normed space X into a normed space Y is itself a normed space with norm defined by $\|T\| = \sup_{x \in X,\, \|x\| = 1} \|Tx\|$.

    (b) If Y is a Banach space, prove that B(X, Y) is a Banach space.

14. State and prove Parallelogram equality. Using parallelogram equality, prove that the space ℓ^p with $p \neq 2$ is not a Hilbert space.

15. State and prove Gram-Schmidt process.

16. State and prove Generalized Hahn Banach theorem.