K25P 2898

Reg. No. : .....

Name : .....

III Semester M.Sc. Degree (C.B.C.S.S. – OBE – Reg./Supple./Imp.) Examination, October 2025
(2023 Admission Onwards)
MATHEMATICS/MATHEMATICS (MULTIVARIATE CALCULUS AND MATHEMATICAL ANALYSIS, MODELLING AND SIMULATION, FINANCIAL RISK MANAGEMENT)
Open Elective Course
MSMAT03O01/MSMAF03O01 : Graph Theory

Time : 3 Hours Max. Marks : 80

PART – A

Answer **any five** questions from this Part. **Each** question carries **4** marks. **(5×4=20)**

1. Define a simple graph. Show that if G is simple, then $\varepsilon < {}^{v}C_2$.
2. Explain the incidence matrix of a graph G with an example.
3. If G is hamiltonian then, show that for every nonempty proper subset S of V, $\omega(G - S) \leq |S|$.
4. Define chromatic number of a graph G. Prove that $\chi(K_n) = n$.
5. State Vizing's theorem and Hajo's conjecture.
6. Prove that a graph G is embeddable in the plane if and only if it is embeddable on the sphere.

PART – B

Answer **any three** questions from this Part. **Each** question carries **7** marks. **(3×7=21)**

7. Show that if there is a (u, v)-walk in G, then there is also a (u, v)-path in G.
8. Prove that a connected graph is a tree if and only if every edge is a cut edge.

P.T.O.

9. Prove that a nonempty connected graph is Eulerian if and only if it has no vertices of odd degree.

10. Let G be a connected graph that is not an odd cycle. Then prove that G has a 2-edge colouring in which both colours are represented at each vertex of degree at least two.

11. State and prove the Euler's formula for planar graphs.

PART – C

Answer **any three** questions from this Part. **Each** question carries **13** marks. **(3×13=39)**

12. Let G be connected with $v > 3$. Show that
    a) if G has a cut edge, then G has a vertex v such that $\omega(G - v) > \omega(G)$
    b) the converse of (a) is not necessarily true.

13. For any graph G, prove that $k \leq k' \leq \delta$.

14. If G is a k-regular bipartite graph with $k > 0$, then prove that G has a perfect matching.

15. Define subdivision of a graph G. If G is 4-chromatic, then prove that G contains a subdivision of K_4.

16. For any two integers $k \geq 2$ and $l \geq 2$, prove that $r(k, l) \leq r(k, l-1) + r(k-1, l)$, where r(k, l) are the Ramsay numbers.

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